An Approach to Predicting Passenger Operation Performance from Commuter System Performance

Bo Chang, Ph. D
SYSTRA
New York, NY

ABSTRACT

In passenger operation, one often is concerned with on-time performance. On-time performance in commuter operation is measured by the difference between the actual arrival or departure time and scheduled arrival or departure time, respectively. A common practice is to use a time threshold value, and measure the schedule deviation. If the schedule deviation is greater than the set threshold value, the operational performance is considered to have failed. Otherwise, the operational performance is considered successful.

At the time a commuter system is commissioned, the system delivers its predicted performance based on its component and subsystem performance, usually expressed in Reliability, Availability, and Maintainability (RAM) measures. The Reliability is usually expressed in Mean Time Between Failures (MTBF) for ground based equipment, and Mean Distance Between Failures (MDBF) for onboard equipment. System failures, however, do not necessarily cause schedule deviation, and if they do, they do not necessarily cause a schedule deviation above the preset threshold value. This paper presents a method that can be used to predict passenger operation performance measured in schedule deviation with a preset threshold value, from delivered commuter system performance expressed in MTBF or MDBF. When used in procurement, this method lends commuter railroads a means to specify system performance in accordance with the target passenger operation performance. For systems already in service, this method allows a commuter railroad to make selective improvements that bring the highest return on investment.

BACKGROUND

A commuter system provider delivers the system with predicted RAM parameters. RAM parameters include MTBF, Mean Time To Repair (MTTR), and availability, A, which can be calculated with the following formula: $A = \frac{MTBF}{MTBF+MTTR}$. (Note: alternately the availability can be defined with Mean Down Time (MDT) that is the sum of MTTR and Mean Travel Time (MTT)). One frequently used method for reliability prediction is to collect component MTBF values, either from the past service data, or using a model, such as the reliability model described in MIL-STD-756B [1], and MTTR, usually derived from a maintenance concept. The MTBF and MDBF have a simple relationship if an average train speed is available. The commuter system performance measures component failures that cause loss of system function regardless whether the system is in passenger service operation or not.

A commuter railroad provides passenger services, most often, based on a schedule. On-time performance is defined as a schedule deviation, or the time difference between actual arrival or departure time and scheduled arrival or departure time, respectively. This schedule deviation is further classified as tolerable if the length falls below a preset threshold value. All equipment failures inevitably require repairing and counted as system failures. These failures, however, do not necessarily cause schedule deviation, and if they do, they do not necessarily cause a schedule deviation above the preset threshold value. As a result, not all system failures can be counted as passenger operation failure.

A commuter system vendor delivers a system with predicted and demonstrated MTBF and/or MDBF, MTTR, and availability, not directly in the passenger operation performance expressed in a schedule deviation with a preset threshold value. This paper presents a statistical way to predicting passenger operation performance from given commuter system performance. When used in procurement, this method lends commuter railroads a means to specify system performance in accordance with the target passenger operation performance. A second potential use of this method is for a commuter operator to make targeted commuter system improvements that directly and positively impact the passenger operation performance resulting maximized return on investment.

THEORY OF THE APPROACH

The described approach uses basic probability theory [2] as briefly summarized below. A set Ω, called the sample space, representing the collection of possible outcomes of a random experiment. An event (denoted in capital letters) A is defined as a subset of the sample space, that is, a collection of points of the sample space. A particular event is the impossible event denoted by the empty set Φ, meaning it will never occur in the given performance of the experiment. The basic algebra of events includes union, intersection, and complement. The union of events A and B, denoted by $A \cup B$, is defined as the set consisting of those points belonging to either A or B (in either one of A or B, or contained in both A and B). The intersection of events A and B, denoted by $A \cap B$, is defined as the set of points that belong to both A and B. The complement of A, written A^c, is defined as the set of points in the sampling space Ω that do not belong to A.

The probability of event A, denoted by $P(A)$, is a number between 0 and 1, with $P(\Omega) = 1$ and $P(\emptyset) = 0$. If A and B are disjoint events, $(A \cup B) = P(A) + P(B)$ (a.k.a. the addition rule). We further define two events A and B are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$ (a.k.a. the multiplication rule). This definition can be extended to multiple events in the following way: Let $A_i, i \in I$, where I is an arbitrary index set, possibly infinite, be an arbitrary collection of events, the A_i are said to be independent if and only if for each finite set of distinct indices $i_1, \cdots, i_k \in I$ we have $P(A_{i_1} \cap A_{i_2} \cap \cdots \cap A_{i_k}) = P(A_{i_1}) \cdot P(A_{i_2}) \cdots P(A_{i_k})$.

Next, we will formulate a probability model for the passenger operation with a commuter system. First, the passenger operation can be viewed as a collection of commuter system functions in execution. This collection of functions can be mathematically expressed as a set, $\{f_i\}$ where f_i is a system function executed in passenger operation. The execution of these functions, in general, occurs in single, or a subset, but not all at all the time. This unevenness in execution, can be expressed in execution frequency, or probability of execution for each system function, P_{f_i}, the probability for function f_i to be in execution. A constraint from the basic probability theory yields $P(\{f_i\}) = 1$, meaning while the commuter system is in service, there is always one or more function, f_i, being executed. With this model, we can draw the following conclusion. Given the fact that not all system function failures cause schedule delay, or schedule delay exceeding the preset threshold value, it is expected that probability of function failures causing schedule delay exceeding the preset threshold value is less than 1, or mathematically, $P(\{f_i\}) < 1$, where the failure of f_i will cause schedule delay exceeding the preset threshold value. Strictly speaking these functions are not necessarily independent. For the purpose of this analysis, we assume all chosen functions are independent.

Second let us examine how system component failures cause a passenger operation functional failure. A given system function f_i in a commuter system is usually performed by one or more system component, $\{C_j\}$ where C_j is a system component performing function f_i with failure rate λ_j, where $\lambda_j = \frac{1}{MTBF}$. The system component failures for a given function may not necessarily be independent among them, or C_j. For the purpose of this analysis, we assume they are, the result hence is the upper bound. Thus, according to the probability formula for disjoint events, the probability for f_i to fail can be expressed as $\sum_{C_j} \lambda_{ji}$, where $\lambda_{ji} = \begin{cases} \lambda_j \text{ if component } C_j \text{ performs function } f_i \\ 0 \text{ if } C_j \text{ does not perform function } f_i \end{cases}$. In addition, it is a reasonable assumption that the event of a function f_i is in execution, and the event that a component performing function f_i has failed, are two independent events, from the basic probability theory, we can say that the probability of function f_i fails while in use is given by $\left(\sum_{C_j} \lambda_{ji}\right) \cdot P_{f_i}$.

Third, we will derive an equation for passenger operational failure, when schedule delay exceeds the preset threshold value, caused by a commuter system component failure. In a commuter system, such as Amtrak Advanced Civil Speed Enforcement System (ACSES), there is defined backup operation to mitigate the impact to passenger operation due to system failures. In Amtrak ACSES system, there are three layers. The top layer of operation is the ACSES, the middle layer is a Cab Signaling system (alternately, these two layers can be viewed as in parallel), and the bottom layer is the Centralized Traffic Control (CTC) system (this is true in most track areas). In case that the ACSES loses its function, the Cab Signaling without ACSES operates. The Cab Signaling can mitigate ACSES failure caused passenger operation performance degradation. In case that both the ACSES and Cab Signaling fail, the CTC operates without ACSES or Cab Signaling. And the CTC can mitigate failures of ACSES and Cab Signaling, which cause passenger operation performance degradation. In other words, not all ACSES failures will result in schedule delay; and when schedule delay does occur, the delay may not necessarily exceed the preset threshold value. Therefore, given a commuter system with deployed backup operation(s), the failure of the primary operation caused by a system function f_i can be mitigated with certain probability, expressed as P_{m_i}, by the backup operation. The author of this paper argues it is a reasonable assumption that the following three events are independent: a system component failure, a system function f_i is in execution, and the backup operation mitigation function is in good operation. Thus, the basic probability theory yields that the probability of schedule delay exceeding the preset threshold value caused by system component failure(s) affecting function f_i with backup operation is: $\left(\sum_{C_j}\lambda_{ji}\right)\cdot P_{f_i}\cdot P_{m_i}$. And the total predicted passenger operation performance is bounded by $\sum_{i=1}^{n}\left[\left(\sum_{C_j}\lambda_{ji}\right)\cdot P_{f_i}\cdot P_{m_i}\right]$, where n is the number of functions when failures would cause schedule delay exceeding the preset threshold value. This is the actual probability when the use of functions f_i is independent from each other.

Last, we will rearrange the terms in the above expression in a second useful form through pure mathematical manipulation as following. $\sum_{i=1}^{n}\left[\left(\sum_{C_j}\lambda_{ji}\right)\cdot P_{f_i}\cdot P_{m_i}\right]=\sum_{1}^{n}\left[\left(\sum_{C_j}\lambda_{ji}\cdot P_{f_i}\cdot P_{m_i}\right)\right]$ $=\sum_{C_j}\sum_{i}^{n}\left(\lambda_{ji}\cdot P_{f_i}\cdot P_{m_i}\right)=\sum_{C_j}\lambda_{ji}\sum_{1}^{n}P_{f_i}\cdot P_{m_i}$, which is the Expression (1).

DESCRIPTION OF THE METHOD

The proposed approach uses the above derived formula to estimate the operation failure probability. In this section, the author suggests one method to estimate the parameter values used in the formula. Once the parameter values are in place, the computation for predicted passenger operation performance is a trivial mathematics problem. The overall approach is depicted in Figure 1. Each step is further detailed in the remainder of this section.

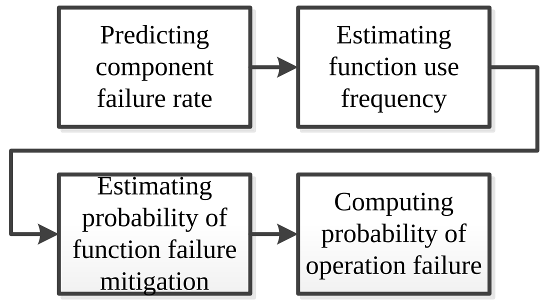

Figure 1.Method for Estimating Passenger Operation Failure Probability

Predicting the Component Failure Rate λ_j

The failure rate λ_j for each component C_j can be either from the past service data in the same or similar equipment for the same or similar application in the same or similar environment, Part Stress Analysis, or Parts Count as described in MIL-HDBK-217 [3]. The Part Stress Analysis and Parts Count method are long practiced and well understood. The author will omit it. The following is a brief summary from [4] for deriving failure rate λ_j from the past service data or lab tests.

Suppose that we have n identical components (C_j) in operation at the same time. We first derive the probability that r or less failures occur assuming the reliability of each component is p. This is the probability for $0,1,2,\cdots,r$ failures to occur. Let $q=1-p$, then for n component devices we have $1=(p+q)^n$. Its binomial expansion is $1=\sum_{k=0}^{n}\left(\frac{n!}{k!(n-k)!}\right)p^{n-k}q^k$. The probability for $0,1,2,\cdots,r$ failures to occur is $p(r\ or\ fewer\ failures)=\sum_{k=0}^{r}\left(\frac{n!}{k!(n-k)!}\right)p^{n-k}q^k$. By Poisson Approximation Theorem (see [5] for the theorem and proof) with large n and small q, $\left(\frac{n!}{k!(n-k)!}\right)p^{n-k}q^k\approx\frac{(nq)^k}{k!}e^{-nq}$. For the component device in consideration, from reliability theory, we can assume that $p=e^{-\lambda t}$, where t is the observation time (test time duration). For small λt, $=1-p=1-e^{-\lambda t}\approx\lambda t$. Thus $p(r\ or\ fewer\ failures)\approx\sum_{k-0}^{r}\frac{(nq)^k}{k!}e^{-nq}$, substitute λt for q, $p(r\ or\ less\ failures)\approx\sum_{k=0}^{r}\frac{(n\lambda\ t)^k}{k!}e^{-n\lambda}$, and $1-p(r\ or\ fewer\ failures)=1-\sum_{k=0}^{r}\frac{(n\lambda t)^k}{k!}e^{-n\lambda t}$, which is the Expression (2).

Now let us turn the attention to the Chi-square distribution table. The mean of the Chi-square distribution is the degree of freedom and the standard deviation is twice the degree of freedom. The integration of Chi-square distribution gives the cumulative probability $P(\chi^2 \le \tau)$. The value $\alpha = 1 - P(\chi^2 \le \tau)$ is a.k.a. the level of significance (and 1-α is the so called confidence). Mathematically, $(\chi^2 \le \tau) = 1 - e^{-\frac{\tau}{2}} \sum_{k=0}^{r} \frac{1}{k!} \left(\frac{\tau}{2}\right)^k$. Compare this expression with the above expression (2), we see they match with $\tau = 2n\lambda t$. More specifically, we compare first few values for failure numbers $r = 0, 1, 2, 3, and\ 4$ with the first five values from the Chi-square table for the degree of freedom values $2, 4, 6, 8, and\ 10$ computed from its density function (see **Error! Reference source not found.**, where F(r) is the number of observed failures, df is the degree of freedom in Chi-square distribution). In general, we have $df = 2r + 2 = 2(r + 1)$.

F (r)	$1 - \sum_{k=0}^{r} \frac{(n\lambda t)^k}{k!} e^{-n\lambda t}$
0	$1 - e^{-n\lambda t}$
1	$1 - e^{-n\lambda t}(1 + n\lambda t)$
2	$1 - e^{-n\lambda t}\left(1 + n\lambda t + \frac{(n\lambda t)^2}{2!}\right)$
3	$1 - e^{-n\lambda t}\left(1 + n\lambda t + \frac{(n\lambda t)^2}{2!} + \frac{(n\lambda t)^3}{3!}\right)$
4	$1 - e^{-n\lambda t}\left(1 + n\lambda t + \frac{(n\lambda t)^2}{2!} + \frac{(n\lambda t)^3}{3!} + \frac{(n\lambda t)^4}{4!}\right)$

Table 1. Probability with Failures in n Components

df	$1 - e^{-\frac{\tau}{2}} \sum_{k=0}^{r} \frac{1}{k!} \left(\frac{\tau}{2}\right)^k$
2	$1 - e^{-\frac{\tau}{2}}$
4	$1 - e^{-\frac{\tau}{2}}\left(1 + \frac{\tau}{2}\right)$
6	$1 - e^{-\frac{\tau}{2}}\left(1 + \frac{\tau}{2} + \frac{\left(\frac{\tau}{2}\right)^2}{2!}\right)$
8	$1 - e^{-\frac{\tau}{2}}\left(1 + \frac{\tau}{2} + \frac{\left(\frac{\tau}{2}\right)^2}{2!} + \frac{\left(\frac{\tau}{2}\right)^3}{3!}\right)$
10	$1 - e^{-\frac{\tau}{2}}\left(1 + \frac{\tau}{2} + \frac{\left(\frac{\tau}{2}\right)^2}{2!} + \frac{\left(\frac{\tau}{2}\right)^3}{3!} + \frac{\left(\frac{\tau}{2}\right)^4}{4!}\right)$

Table 2 Chi-square Cumulative Probability with r Degree of Freedom

Thus, for observed 3 failures among n deployed devices in given time t, we would use a Chi-square table value for degree of freedom $df = 2(r + 1) = 8$.

In order to use the Chi-square table for component device failure rate calculation, we need to keep the following in mind:

1. Set the degree of freedom to 2(r+1)
2. Assign (determine) a confidence level, 1-α
3. Determine χ^2_α such that $P(0 \le \tau \le \chi^2_\alpha) = 1 - \alpha$, or $P(0 \le 2n\lambda t \le \chi^2_\alpha) = 1 - \alpha$. And equivalently $P\left(0 \le \lambda \le \frac{\chi^2_\alpha}{2nt}\right) = 1 - \alpha$, where n is the number of deployed devices (or elements in test) and t is the observation time (or test time duration).

Estimating the Function Use Frequency P_{f_i}

In this section, the author describes a method to estimate the probability, or the frequency, of a commuter system function that is in use during passenger service operation, P_{f_i}. This method has the steps as shown in Figure 2.

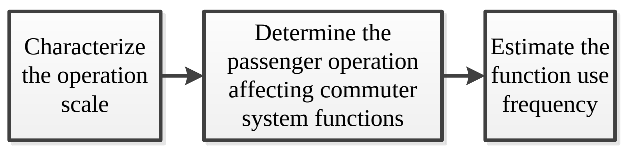

Figure 2.Method for Estimating P_{f_i}

Characterize the operation scale

The operation scale varies. In general, this should include the number of trains per hour running in the system in a period of concern, number of trains per hour that a stationary wayside equipment services, and the fraction (specific components, C_j) of the onboard subsystem, which is use. The fraction of wayside subsystem in use is measured by the number of trains per hour been serviced by the wayside equipment. And the fraction of the office subsystem in use is measured by the operation hours in each day.

The author will mention two commonly used measures for the number of trains per hour running in the system, one is the peak service and the other is the average service. First, a peak number of trains in service per hour and an average number of trains in service per hour are estimated. These two numbers can come from the current operation schedule, if no planned schedule change, or from a planned operation schedule. Second, for wayside stationary train service equipment, such as a wayside control point or a communications base station, a peak number of trains been serviced by the stationary wayside equipment per hour and an average number of trains that are serviced by the stationary wayside equipment per hour are estimated. Again, this estimate can be from the current operation schedule if no planned schedule changes, or a planned schedule. Because these two numbers are location dependent, for example, on Caltrain mainline, San Francisco station at the fourth street has the most number of trains per hour, the computation can be divided in multiple segments for improved accuracy. For simplicity, this paper will assume a uniform distribution of trains per hour throughout the concerned areas. Third is to determine the fraction of the onboard commuter subsystem in use. For example, in an Electrical Multiple Unit (EMU) consist of four married-pairs, there would be four sets of the commuter subsystem equipment, and only one is in control. In this case, the use factor for the onboard subsystem is 0.25 on each train.

Determine the passenger operation affecting commuter system functions

From the commuter system architecture and design information, system concept of operations, a set of commuter system functions that impact the passenger operation performance is identified. For example, in the Caltrain's Communication Based Overlay Signal System (CBOSS), the function of location determination impacts the passenger operation performance.

Estimate the function use frequency

For each identified commuter system function that impacts passenger operation performance, estimate its use frequency. This use frequency in general depends on the operation scale, commuter system technology, and the railroad infrastructure. It is not feasible nor the author's intention to comprehensively cover all variations. In this paper, however, the author will present the following main aspects: transponder reading function as used in CBOSS and the Federal Railroad Administration (FRA) Type Approved Advanced Civil Speed Enforcement System (ACSES); wayside status query as used in ACSES, CBOSS, and Electronic Train Management System (ETMS); and Temporary Speed Restriction (TSR) query as used in ACSES.

Transponder reading function. This function is in use when a train passes over a transponder. The use frequency can be estimated by the ratio of the reading time duration over the train running duration in the interested railroad segment or area.

Wayside status query. This function is in use when a train starts querying the wayside. The use frequency can be estimated by the ratio of the querying time duration over the train running duration in the interested railroad segment or area.

TSR query. This function is in use when a train starts querying the TSR. In ACSES, this is serviced by the Safety TSR Server (STS). The use frequency can be estimated by the ratio of the querying time duration over the train running duration in the interested railroad segment or area.

Estimating the Probability of Function Failure Mitigation P_{m_i}

Next is to estimate the probability that a failed function can be mitigated, or the mitigation P_{m_i} associated with f_i. For each identified commuter system function that impacts passenger operation performance, estimate the probability that such a function failure can be mitigated in the given commuter system with the given operation procedures and maintenance concept, or MTTR statistics. This mitigation probability in general depends on the commuter system technology, and the maintenance concept. For example, in CBOSS or ACSES, a failed automatic function may often be mitigated with the underlying signaling system, such as Cab signaling, Centralized Traffic Control (CTC), or Automatic Block Signaling (ABS), in conjunction with a maintenance policy that the failed onboard equipment will be sent directly to a maintenance shop. Again, it is not feasible nor the author's intention to comprehensively cover all variations. In this paper, however, the author will present the following main aspects: transponder reading function; wayside status query; and TSR query.

Transponder reading failure mitigation

When a ground transponder or onboard transponder reading function fails, different commuter systems have different mitigation means. In FRA Type Approved ACSES system, when a permanent transponder set is either completely missed or partially missed, the ACSES will purge all currently active transponder permanent speed restrictions, and will enforce the lesser of the maximum vehicle speed, TSR, and route dependent speed restriction. The same time, the ACSES will alert the train crew of the situation and train crew usually will be able to complete the scheduled run. In worst case scenario, the engineer will cut out ACSES and complete the scheduled run with Cab signaling, CTC, or ABS. The value of this probability can be estimated from the past operation statistics or through analysis.

Wayside status query failure mitigation

The locomotive queries the wayside equipment periodically. If the wayside equipment, wayside communication equipment, or onboard equipment fails, the operation will fall back to Cab signaling, CTC, ABS, or other backup operations designed in the commuter system while restoration is in progress. The backup operation usually depends on the train crew that introduces possibility of human errors, but usually is as efficient as without the failure. As a result, the onboard failures can be effectively mitigated. Again the value of this probability can be estimated from the past operation statistics.

TSR query failure mitigation

Next is to identify the locomotive queries the office through wayside communication periodically for TSR updates. If the wayside communication, office, or onboard equipment fails, the operation will fall back to the established backup operation. In most cases, this is a radio based manual procedure. The TSR is usually preplanned, for example, in the previous day. As a result, most often, the TSR information does not change during the daily operation. This means that the probability such system failure will be effectively mitigated is quite high.

Computing Probability of Operation Failure

This computation can use Expression (1) to generate a table or Spreadsheet. Each row in the table is a commuter system component C_j with its predicted failure rate λ_j. Associated with each system component, there is a list of passenger operation functions with its use frequency and function failure mitigation. Summing up $\sum_1^n P_{f_i} \cdot P_{m_i}$ for each component yields the value when the use of these functions are independent from each other, or the upper bound otherwise.

This value of $\sum_1^n P_{f_i} \cdot P_{m_i}$ indicates the weight of component C_j in passenger operation performance, larger the value, more weight of the component. Reducing the failure rates of the most significant components produces the highest cost benefit return. For a newly acquired system, these components should be managed with more rigor, if managing all rigorously is not cost effective. In case of making improvement over an in-service system, these components should be considered first.

CONCLUSION

An approach to predicting passenger operation performance from commuter system performance has been developed. This method produces actual values if all the parametric events are independent. In case, some or all are dependent, this method yields an upper bound to the weight of the component with respect to passenger operation performance.

With operation performance affecting weight calculated for each component of a commuter system, it is possible for a system supplier or an acquirer to focus on a selected few components in case giving equal focus to all system components is either not cost effective or feasible. For an existing commuter system, the owner may focus on improving the selected few components and achieve the most return on investment.

Future study may look into passenger operation functions together with their correlation so that the derived expressions may be refined for its accuracy or applicability.

REFERENCES

1. MIL-STD-756B, Military Standard, Reliability Modeling and Prediction, Department of Defense, United States of America, November 18, 1981.
2. Chapter 1 – Basic Concepts of Basic Probability Theory, Robert B. Ash, Department of Mathematics, University of Illinois, at http://www.math.uiuc.edu/~r-ash/BPT/BPTCh1.pdf.
3. MIL-HDBK-217F, Military Handbook, Reliability Prediction of Electronic Equipment, Notice 2, December 2, 1991.
4. Vito Faraci Jr., Measuring Failure Rate by Testing, The Journal of the Reliability Information Analysis Center – First Quarter 2010.
5. http://en.wikipedia.org/wiki/Poisson_limit_theorem