Fare Elasticity and Its Application to Forecasting Transit Demand

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APTA

FOREWORD

Fare Elasticity and Its Application to Forecasting Transit Demand represents the first comprehensive effort to estimate the fare elasticities of a large number of transit systems using monthly data, and to test the applicability of the well known Simpson-Curtin formula in today's transit environments.

The study provides a general approximation of system-wide bus ridership loss following a uniform fare increase, that is without changing the fare structure. It is not intended to replace detailed fare elasticity estimates conducted for specific transit systems.

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The analysis shows that the impact of fare changes on bus ridership, while varying substantially among cities and between peak and off-peak hours, is more pronounced than previously believed.

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ABSTRACT

Transit managers are under increasing pressure to obtain sufficient fare revenues to maintain superior service while reducing dependence on government assistance. They need an accurate formula to estimate the impacts of fare changes on transit ridership and fare revenues. For years, these managers were given two choices: constructing a fare elasticity model specific to their transit systems <u>or</u> applying the Simpson-Curtin formula which postulates a fare elasticity of -0.33; i.e., a 10 percent increase in fare would result in a 3.3 percent decrease in transit patronage.

The models are usually costly and time-consuming to construct, causing delays in the implementation of fare changes. On the other hand, the 30-year-old Simpson-Curtin formula is likely to be inaccurate today. Further, it provides no estimation of the varying fare impacts between peak and off-peak hours, or between large and small cities.

The objectives of this study are to verify the Simpson-Curtin formula using updated data and modern technologies, and to provide a set of fare elasticity estimates for bus service in various cities during peak as well as off-peak hours.

An advanced econometric model, the Autoregressive Integrated Moving Average (ARIMA) model, was used for the estimations. A special survey was conducted to obtain ridership data 24 months before and 24 months after each fare change for 52 transit systems. Monthly information on other factors which may influence ridership, including gasoline price, vehicle miles of service, labor strikes, etc., were also collected. The purpose was to use the model to isolate the impacts of the fare changes from those caused by other factors.

Findings

- On the average, *a ten percent increase in bus fares would result in a four percent decrease in ridership*. This shows that today's transit users react more severely to fare changes than found by Simpson and Curtin.

FARE ELASTICITY - BUS SERVICES	
Average (all hours, all cities)	-0.40

- Transit riders in small cities are more responsive to fare increases than those in large cities. *The fare elasticity for bus service is -0.36 for systems in urbanized areas of 1 million or more population. In urbanized areas with less than 1 million people, the elasticity is -0.43.*

- Although the data for peak vs. off-peak services are available for only six transit systems, the difference between the fare elasticity levels is very clear: *The average peak-hour elasticity is -0.23 while the off-peak hour elasticity is -0.42,* indicating that peak-hour commuters are much less responsive to fare changes than transit passengers travelling during off-peak hours.

FARE ELASTICITY - BUS SERVICES

	Cities/Areas with Population of	
	more than 1 million	less than 1 million
All hour average	-0.36	-0.43
Peak hour average	-0.23	
Off-peak average	-0.42	
Peak hours	-0.18	-0.27
Off-peak	-0.39	-0.48

EXECUTIVE SUMMARY

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Fare elasticity measures the response of transit patronage to fare changes. In a simple mathematical sense, it is defined as the ratio of percentage change in ridership to a one percent change in fare. For example, if a one percent increase in fare results in a half percent decrease in ridership, the fare elasticity is -0.5. The negative sign (-) indicates that fare and ridership move in opposite directions. If the absolute value of fare elasticity is greater than 1 (e.g., elasticity = -1.2), any increase in fare would cause a larger decline in ridership, resulting in a decrease of total fare revenue. Alternatively, an absolute fare elasticity of less than 1 implies that a fare increase will result in increased revenues. Knowledge of fare elasticity is extremely important for transit managers, as it provides information on the expected ridership and farebox revenue resulting from a proposed fare change.

The impact of fare on transit ridership has been an unsettled issue for many decades. While it generally is recognized that a fare increase would result in some ridership decrease, the magnitude of such decrease is difficult to measure and can vary greatly among transit systems. The problem stems from the fact that ridership does not respond to fare changes immediately. However, over a longer time period, the observed ridership changes may be caused by factors other than the fare change, resulting in an erroneous elasticity estimation.

Dozens of fare elasticity studies have been completed in the past decades. Some suffer from serious analytical shortcomings rendering the results questionable. Others are either overly complicated or overly specific to individual transit properties. For example, fare elasticities are commonly estimated on specific routes for specific transit systems. The results cannot be generalized, and the usefulness of the studies are limited to the particular situations for which the studies are designed. As a result, most smaller and medium-size transit operators with limited research resources have often made important fare decisions based on a simple rule of thumb which assumes a fare elasticity value of -0.33 for all transit routes during all times of day. This method, commonly referred to as the Simpson-Curtin formula[1], is inadequate to meet the information needs for determining fare policies.

This study attempts to establish a fare elasticity estimation procedure that preserves the Simpson-Curtin simplicity while using the econometric methods and computer technology of the 1990s. The purpose of this study is two-fold. First, it develops an advanced econometric model, **the transfer function model**, to be applied by transit systems in estimating fare elasticities. Secondly, the results are used to search for a pattern of fare elasticity behavior which enables those transit systems without a modeling capability to arrive at

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[1]John F. Curtin, "Effect of Fares on Transit Riding," *Highway Research Record*, 213 (1968), 8-19.

an approximate elasticity estimate by using those of similar systems. To accomplish these purposes, the fare elasticities of a sample of fifty-two transit systems are estimated, with six systems having the elasticities broken down to peak and off-peak hours. The sample is selected such that transit systems of different sizes, serving large cities as well as small rural areas are represented. Clearly, this method is not as desirable as applying the model directly for elasticity estimation. However, it is superior to indiscriminate use of the Simpson-Curtin rule of thumb.

METHODOLOGY

Overview

Popular methods used for estimating transit fare elasticity may be divided into three broad categories:

- Preference Survey
- Shrinkage Analysis
- Econometric Studies

Preference Survey. Surveys are conducted to obtain information on the intended modes of travel under various conditions. For example, survey respondents are asked if they intend to commute to work by car or transit if the bus fare is raised by 25 cents, waiting time averages 10 minutes, and the parking cost is $60 per month. With a large number of responses on similar questions, it is possible to statistically estimate the relative importance of the fare, service attributes and other transportation factors to determine the fare elasticities for various market segments.

A major shortcoming of this approach is that the respondents' intentions may and usually do differ from the actual events. A Chicago study[2] found that this method resulted in high elasticity estimates because individuals responding to the questionnaire had assumed that a car would be available for their journey, whereas in practice this was not always the case.

Shrinkage Analysis. This approach measures fare elasticities by monitoring the ridership levels prior to and after a fare change. Fare elasticity is estimated by computing the ratio of the percentage change in ridership to percentage change in fare.

This method is simple, but may not provide accurate results because of unavoidable outside interferences. For example, if a transit authority raises fare on June 1, the observed decrease in ridership may also be caused by fewer student riders as the summer vacation begins. Taking the ratio of ridership change to fare change between May and June would capture the effects of both the fare change and the school year seasonality, resulting in an erroneous fare elasticity estimate. The June ridership may be compared to the previous year's June ridership to avoid

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[2] C. Phillips Cummings, et al., "Market Segmentation of Transit Fare Elasticities", *Transportation Quarterly*, XLIII (July 1989), 418-419.

seasonal bias. The results of this comparison could also be misleading since other factors, such as changes in gas prices and transit service, may have influenced ridership during this twelve month span.

Econometric Studies. Most popular among this group is regression analysis, which uses historical data to estimate the demand function for transit patronage. Econometrics allows the relationship between ridership and its influential factors such as fare, time of day, trip purposes, cost of alternative modes, and socio-economic characteristics of the population to be expressed in mathematical forms. The effects of fare changes on transit patronage can then be isolated to arrive at unbiased fare elasticity estimates. The model may be a *cross sectional analysis* which uses data over many geographic areas for a given point in time, or *a time series analysis* which models the variation of fare and demand over time.

From a theoretical standpoint, the time series analysis is a preferred method. The *cross sectional analysis*, which does not consider the effects of time, may not adequately capture the responses of individuals or cities in response to fare changes over time. Rather, it reflects how different population segments behave at different fare levels. For lack of better terms, cross-sectional models are often considered as an indication of Àlong run adjustments.Â Thus, although cross-sectional estimates have some advantage in forecasting structural changes in demand, it cannot be used to measure the short run responses of ridership to fare changes with a reasonable degree of confidence unless supporting time series information is available. Nevertheless, data limitations frequently necessitate independent use of cross-sectional analysis in fare elasticity research and ridership forecasting.

On the other hand, traditional *time series analysis* also suffers drawbacks. This approach commonly involves regression analysis in which the ordinary least square (OLS) method is used to fit transit demand functions. Several crucial assumptions, including one requiring no serial correlation present in the error term, are usually violated, rendering the estimated transit demand function biased and the fare elasticities unreliable. This well-known autocorrelation problem has been an unresolved issue facing researchers for decades.

The Transfer Function Model

The present study is a time series analysis. However, it eliminates the autocorrelation and other methodological deficiencies by applying the transfer function model to estimate the transit demand function and fare elasticities. This model is an extended version of the Autoregressive Integrated Moving Average (ARIMA) or Box-Jenkins model, made popular among researchers because of the revolutionary advancement of computer technology.

The *transfer function model* represents a substantial improvement over the standard OLS-time series method in two major aspects. First, it eliminates not only the autocorrelation, but also the multicollinearity and inefficient estimates problems which are common in OLS models. Secondly, it allows for a richer dynamic structure in the relationship between the dependent variable (transit demand) and the explanatory variables (fare, services, gas price, etc.). The model is able to isolate the seasonal fluctuation of transit ridership and capture the delayed effects of ridership responses to fare changes.

When the function is expressed in natural logarithm, the coefficient of the fare variable measures the change in ridership in response to fare change which is, by definition, the fare elasticity. Monthly time series data for fifty-two individual transit systems are used to estimate their transit demand functions.

Data Collection

A special survey was conducted to obtain monthly data for four year periods, 24 months before to 24 months after the latest identified fare change date for each transit system. The data requested included monthly ridership, vehicle miles, vehicle hours, basic adult cash fare, and total farebox revenues during peak and off-peak periods. Other information such as work stoppages and variation in peak-hour definitions were also collected. In addition, monthly data were gathered from nationally published sources on local consumer price indexes, gasoline prices, and local employment for use in the model.

In total, 189 survey questionnaires were mailed to transit operators, and 79 were

Table 1. Transfer Function Model Functional Form

The transfer function model takes the following general form:

$$R_t = f\,(SL_{t-k}, FC_{t-k}, AC_{t-k}, MC_{t-k}, I_{t-k}) + e_t$$

where:

R_t = Transit ridership, measured by unlinked transit passenger trips.

SL_{t-k} = The service level, measured by revenue vehicle miles and/or revenue vehicle hours.

FC_{t-k} = Transit cost, measured by average fare, deflated by the local Consumer Price Index.

AC_{t-k} = Cost of major alternative modes, measured by gasoline price, deflated by the local CPI.

MC_{t-k} = Market characteristics or the size of the transportation market, approximated by the number of people employed locally.

I_{t-k} = Intervention factors. These include, where appropriate, work stoppages, gasoline shortages and other abrupt changes. The I variables are given the value of 1 during the event and 0 otherwise.

e_t = Disturbance term

t = Time period

k = Time lag

returned before the internal deadline. The response rate of 42 percent is considered very high for this type of survey as many transit systems do not keep monthly operating data. Twenty-seven returned questionnaires were unusable, resulting in 52 useable questionnaires and a useable response rate of 28 percent.

Model Application

The Time Transfer Function model was applied to 52 transit systems in cities of various sizes, ranging from 51,000 to nearly 10 million in population. In six cases, for which data are available, the transit demand functions were estimated for peak hours as well as off-peak hours.

Generally, the economic behavior of the transit riders is well predicted by the model. The corrected coefficient of determination ($\bar{R}^2$) ranges from 0.51 to 0.97, denoting that more than 50 percent and up to 97 percent of the fluctuation in transit ridership is explained by the model. Twenty-five cases have a $\bar{R}^2$ of 0.80 or higher, and for seven cases, the model is able to explain more than 90 percent of the ridership variations. Figures 1, 2 and 3 depict examples of how the model performs at different $\bar{R}^2$ levels.

The t-statistics indicate that the fare elasticity coefficients are statistically significant at the 90 to 99 percent confidence level.

Figure 1. Actual vs. Estimated Unlinked Passenger Trips: $\bar{R}^2$=0.92 (Denver, Colo.)

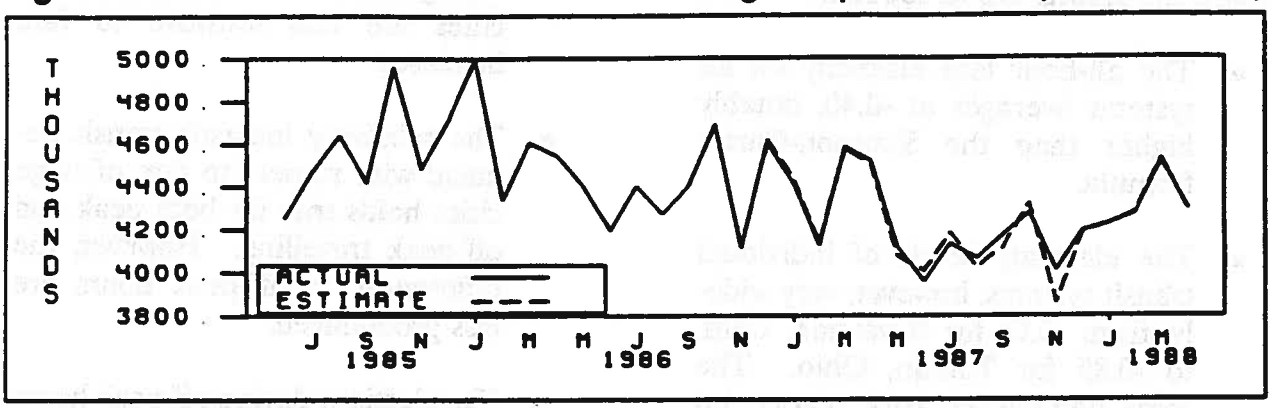


Figure 2. Actual vs. Estimated Unlinked Passenger Trips: $\bar{R}^2$=0.77 (Gretna, La.)

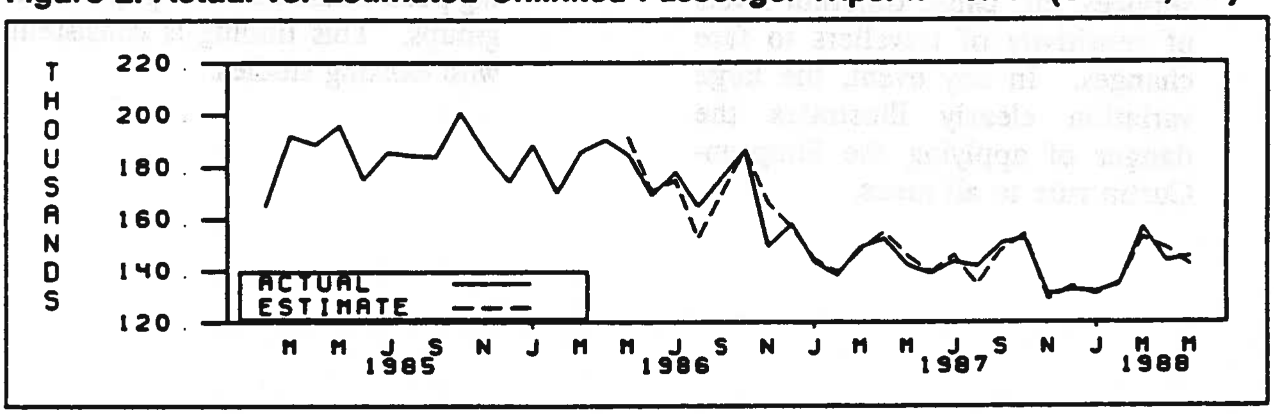
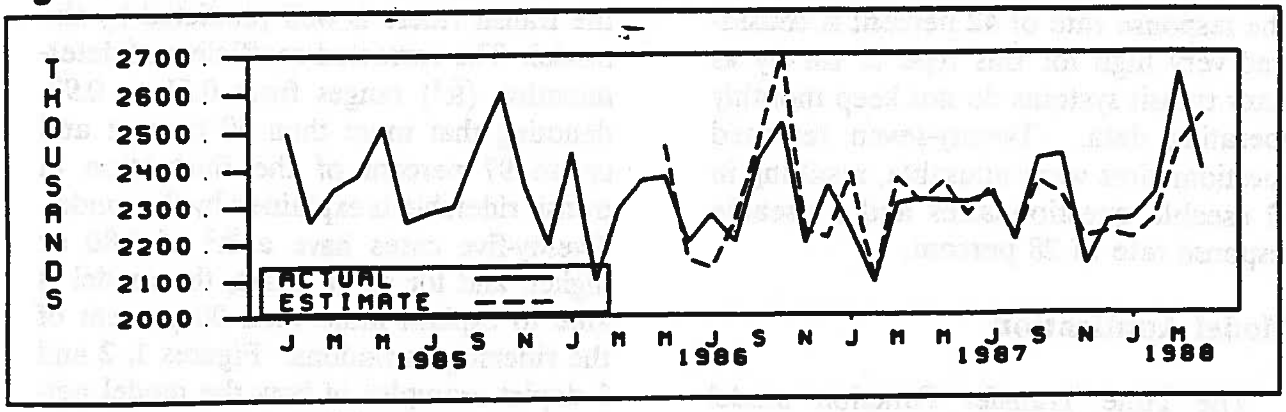


Figure 3. Actual vs. Estimated Unlinked Passenger Trips: $\bar{R}^2=0.52$ (San Jose, Calif.)



RESEARCH RESULTS

The fare elasticities of bus service for fifty-two transit systems under study are presented in Table 2 (all-hour average) and Table 3 (peak/off-peak differential). Briefly, the results are as follows:

- The all-hour fare elasticity for all systems averages at -0.40, notably higher than the Simpson-Curtin formula.
- The elasticity levels of individual transit systems, however, vary widely, from -0.12 for Riverside, Calif. to -0.85 for Toledo, Ohio. The local population work places, income, driving conditions, transit services, etc. cause different levels of sensitivity of travellers to fare changes. In any event, the large variation clearly illustrates the danger of applying the Simpson-Curtin rule to all areas.
- The average elasticity for large cities (more than 1 million population) is much smaller (in absolute value) than the smaller cities, indicating that transit users in large cities are less sensitive to fare increases.
- The relatively inelastic transit demand with respect to fare of large cities holds true for both peak and off-peak travelling. However, the differences in off-peak hours are less pronounced.
- The elasticity during off-peak hours is about twice as high as that during peak hours for both population groups. This finding is consistent with existing studies.

Table 2. Transit Fare Elasticity Estimates of 52 Transit Systems

	CITY	URBAN AREA POPULATION	FARE ELASTICITY	t-STAT	R̄ SQUARED	FARE ELAST GROUP MEANS
	BUS SERVICES IN URBANIZED AREAS WITH MORE THAN 1 MILLION POPULATION					
1	Los Angeles, CA	9,479,436	-0.231	5.83	0.87	
2	Des Plaines, IL	6,779,799	-0.117	1.75	0.73	
3	Detroit, MI	3,809,327	-0.247	3.18	0.92	
4	San Francisco, CA	3,190,698	-0.151	2.28	0.88	
5	Alexandria, VA	2,763,105	-0.412	2.29	0.91	
6	Dallas, TX	2,451,390	-0.134	1.77	0.91	
7	Baltimore, MD	1,755,477	-0.495	3.40	0.78	
8	San Diego, CA	1,704,352	-0.270	1.85	0.66	
9	Oceanside, CA	1,704,352	-0.350	2.64	0.68	
10	Atlanta, GA	1,613,357	-0.277	2.72	0.51	
11	Phoenix, AZ	1,409,279	-0.321	1.86	0.66	-0.361
12	Seattle, WA	1,391,535	-0.266	2.35	0.86	(0.154)*
13	Everett, WA	1,391,535	-0.429	1.82	0.51	
14	Denver, CO	1,352,070	-0.562	20.60	0.92	
15	San Jose, CA	1,243,952	-0.460	2.17	0.52	
16	Cincinnati, OH	1,123,412	-0.738	1.98	0.80	
17	Kansas City, MO	1,097,793	-0.511	4.32	0.92	
18	Gretna, LA	1,078,299	-0.354	3.10	0.77	
19	Portland, OR	1,026,144	-0.387	4.30	0.64	
20	Buffalo, NY	1,002,285	-0.503	3.27	0.84	
	BUS SERVICES IN URBANIZED AREAS WITH LESS THAN 1 MILLION POPULATION					
21	Sacramento, CA	796,266	-0.162	7.58	0.84	
22	Riverside, CA	705,175	-0.119	3.96	0.76	
23	Honolulu, HI	582,463	-0.652	5.99	0.80	
24	St. Petersburg, FL	520,912	-0.478	3.19	0.74	
25	Nashville, TN	518,325	-0.527	3.25	0.82	
26	Richmond, VA	491,627	-0.624	2.43	0.70	
27	Albany, NY	490,015	-0.456	3.42	0.57	
28	West Palm Beach, FL	487,044	-0.605	2.92	0.86	
29	Toledo, OH	485,440	-0.855	29.54	0.97	
30	El Paso, TX	454,159	-0.294	2.54	0.50	
31	Tacoma, WA	402,077	-0.432	4.70	0.63	
32	Allentown, PA	381,734	-0.747	2.60	0.70	
33	Grand Rapids, MI	374,744	-0.430	6.89	0.84	
34	Flint, MI	331,931	-0.585	2.98	0.87	
35	Fresno, CA	331,551	-0.311	4.99	0.74	
36	Sarasota, FL	305,431	-0.214	2.67	0.68	
37	Chattanooga, TN	301,515	-0.341	4.75	0.88	-0.430
38	Spokane, WA	266,709	-0.527	3.15	0.69	(0.189)*
39	Fort Wayne, IN	236,479	-0.116	1.77	0.90	
40	South Bend, IN	226,331	-0.261	4.58	0.66	
41	Madison, WI	213,675	-0.401	2.34	0.83	
42	Eugene, OR	182,495	-0.184	1.89	0.84	
43	Lincoln, NE	173,550	-0.500	3.26	0.55	
44	South Daytona, FL	170,749	-0.423	2.88	0.61	
45	Binghamton, NY	161,132	-0.704	10.95	0.93	
46	Lancaster, PA	157,385	-0.428	2.94	0.79	
47	Appleton, WI	142,151	-0.255	2.86	0.61	
48	Springfield, MO	139,030	-0.481	8.57	0.65	
49	Williamsport, PA	58,650	-0.299	2.52	0.75	
50	Oshkosh, WI	52,958	-0.167	3.09	0.86	
51	State College, PA	51,298	-0.642	4.57	0.89	
52	Boone, NC	Non-UZA	-0.528	5.66	0.81	
	ALL SYSTEMS:					-0.403 (0.179)*

* - Standard Deviation
Source: American Public Transit Association

Table 3. Fare Elasticity: Peak and Off-Peak Travel

Urbanized Area	Peak		Off Peak		Population
Spokane, WA	-0.32		-0.73		266,709
Grand Rapids, MI	-0.29		-0.49		374,744
Sacramento, CA[1]	-0.22		-0.14		796,266
GROUP I AVERAGE[2]	-0.27	[0.04]	-0.45	[0.30]	1 million and less
Portland, OR/WA	-0.20		-0.58		1,026,144
San Francisco, CA[3]	-0.14		-0.31		3,190,698
Los Angeles, CA	-0.21		-0.29		9,479,436
GROUP II AVERAGE[2]	-0.18	[0.04]	-0.39	[0.16]	1 million and more
ALL SYSTEMS AVERAGE[2]	-0.23	[0.06]	-0.42	[0.22]	

Notes: 1. Light rail initiated March 1987, which was during the observation period.

2. The standard deviations of the group and total means are contained in square brackets.

3. Transit system serves Marin and Sonoma counties.

CHAPTER 1

INTRODUCTION

Federal policies to reduce government spending have had a substantial effect on the operating and capital budgets of many public mass transit agencies. With the reduction in federal subsidies, mass transit has had to search for new funding sources. Table 4 shows that as the federal contribution decreases, transit systems receive a larger portion of their operating revenues from state and local governments.

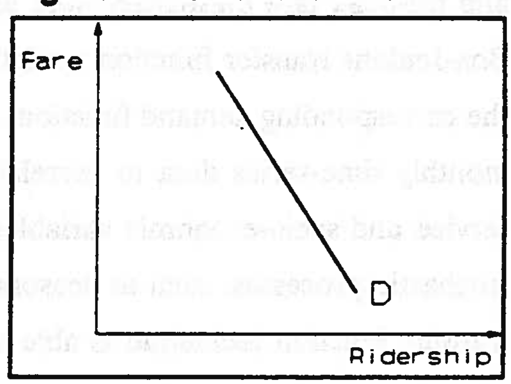
Table 4.
Transit Revenues as Percentage of Total Revenues, All Modes, All U.S. Areas

	1981	1989	% change
Passenger Fares	36.7	36.2	-1.4
Other Operating Revenue	4.6	5.6	21.7
State & Local Operating Assistance	43.8	52.0	18.7
Federal Operating Assistance	14.9	6.2	-58.4

Source: American Public Transit Association, *1991 Transit Fact Book*

Transit systems which raise fares are expected to find that the mass transit industry, like most other goods and services, faces a downward sloping demand curve with respect to price, as shown at the right in Figure 4. The downward sloping demand curve means that as fares increase, ridership will decrease. The shape of this demand curve, steeply curving, gently curving or straight line, indicates the severity of the ridership loss. The passenger reaction to fare change can be quantified by measuring the percent change in ridership occuring with a one percent change in fare. The resultant

Figure 4. Demand Curve



number is known in economics as the ***fare elasticity of demand*** or simply, fare elasticity. Fare elasticity is an important subject to transit planners for four primary reasons:

1. **Ridership and revenue estimation** after a fare increase is an integral part of the transit route scheduling and budgeting processes.

2. There may be serious social and political consequences of increasing bus fares. Therefore, good **fare policy and planning** requires "what if" analysis of passenger behavior.

3. There exists a theoretical point of unitary fare elasticity beyond which increasing fares will result in **decreasing fare revenues** and thereby negate any possible revenue generation through fare policies.

4. **The Simpson-Curtin rule of thumb[3] is over three decades old** and there is empirical evidence that it may not be suitable for small cities or for disaggregated fare analysis.

This study examines the fare elasticity of the fixed-route motor bus operations of fifty-two American transit systems during the years 1982 to 1988. Furthermore, both peak and off-peak fare elasticities were estimated for six of these properties. The ARIMA or Box-Jenkins transfer function model was used to estimate the various fare elasticities and the corresponding demand functions of each individual transit system. This method utilizes monthly time-series data to correlate motor bus ridership with the transit system's fare, service and socio-economic variables. The model also examines the underlying, inherent stochastic processes, such as seasonality, which affect transit ridership. The Box-Jenkins transfer function technique is able to estimate the mathematical relationship or demand

[3] The Simpson-Curtin formula states that ridership will decline by one-third percent for each one percent increase in fare. See John F. Curtin, "Effects of Fares on Transit Riding," *Highway Research Record*, 213 (1968), 8-19.

function between current ridership, the past ridership levels, and other current and past exogenous variables. From these estimated demand functions, individual systems' fare elasticities were calculated.

There have been many transit fare elasticity studies since Simpson and Curtin performed their original work in the 1950s. Using these studies transit planners and economists have attempted to develop a general rule with which to forecast ridership across the transit industry. This paper differs from previous fare studies in the following respects:

- **The scope of the study is very large.** In compiling this study, the all-hours ridership function of 52 transit systems was examined. Additionally, the peak and off-peak hours ridership functions of six transit systems were investigated. These agencies operated large, medium and small fixed-route bus service in every area of the United States. Most other fare studies were conducted or sponsored by transit agencies and focused only that particular system's elasticity.

- **Most fare elasticities were estimated over an analogous forty-eight month observation period in the time space of 1982 to 1988.** In contrast, other researchers have grouped elasticities from fare changes studies which took place as much as 30 years apart.[4] This time similarity lessens the shift in the demand curve which occurs because of changes in technology and consumer attitudes.

- **Since all fare elasticities were estimated using nearly identical statistical procedures, the elasticities presented in this study are directly comparable across transit systems.** Previously, fare elasticity has been estimated by binary choice models, before-after comparisons and cross-sectional analysis, as well as

[4] See Patrick Mayworm, et al., *Patronage Impacts of Changes in Transit Fares and Service*, p. 19 and Appendix A where fare elasticities are aggregated from studies as early as 1948 and as late as 1979.

time-series analysis. Elasticities estimated through application of these different statistical methods within a single transit system seldom concur, usually because each of the methods imposes distinct assumptions and restrictions. Although which method best represents the true demand curve is subject to discussion, the direct comparison of the results of different methods is impossible without vigorous calibration.

- **A practical application of the fare elasticities is presented.** Used correctly, the techniques demonstrated herein will permit the transit planner to estimate ridership loss after a fare change within a certain confidence interval.

Chapter 2 discusses the methodology in more detail. It is designed to prepare the reader with the necessary background to understand how the elasticities were estimated and why the transfer function model was chosen. Chapter 3 presents the results and offers some real world applications. The final chapter demonstrates how to estimate and interpret fare elasticities. The major areas discussed in this paper are:

- *Chapter 2: Theory and Methodology of Model Estimation* is a technical description of the econometrics used to estimate the fare elasticities.

- *Chapter 3: Empirical Testing* demonstrates how the data needed for the fare elasticity analysis was collected. It describes the model applications, elasticity findings and their implications on the effects of fare changes on bus ridership.

- *Chapter 4: Use of the Transfer Function by Transit Systems* discusses the steps required to estimate local fare elasticities and how the results are interpreted.

- *Appendix A: Review of Selected Fare Elasticity Studies* reviews the methods and findings of previous studies on transit fare elasticity.

- *Appendix B: Survey Questionnaire Form* is a sample of the Fare Impact Survey which was sent to 189 transit agencies in May 1988.

- *Appendix C: Equations and Variables* presents the equations estimated by the model in standard regression output format.

- *Appendix D: Bibliography* lists the books, articles and other sources of information consulted during the course of this study.

CHAPTER 2

THEORY AND METHODOLOGY OF MODEL ESTIMATION

Functional Forms of Mass Transit Regression Models

Within a certain transit market at a given point in time *t*, a generalized demand model[5] for public mass transit ridership can be hypothesized to take the following form:

$$R_t = f(SL_t, FC_t, AC_t, MC_t, S_t, I_t) + \epsilon_t \qquad (1)$$

where:

- R_t = mass transit ridership
- SL_t = level of service and accessibility supplied by the transit system
- FC_t = total (fares and time) costs of traveling by mass transit
- AC_t = total costs of traveling by an alternate mode
- MC_t = travel market characteristics including city size and demographics
- S_t = seasonal factors
- I_t = non-periodic interventions such as work stoppages and special promotions
- ϵ_t = random error

The demand function described in equation 1 could be estimated with regression analysis to yield a mathematical model of the form shown in equation 2.

$$\hat{R}_t = \beta_0 + \beta_1 SL_t + \beta_2 FC_t + \beta_3 AC_t + \beta_4 MC_t + \beta_5 S_t + \beta_6 I_t + e_t \qquad (2)$$

In equation 2, all independent variables take their previous definitions, $\hat{R}_t$ is the transit ridership estimate, β_0 through β_6 are the estimated regression coefficients of the corresponding explanatory variables and e_t is the estimated error term, which theoretically should be purely random, but may not be random because the model is a simplification of reality and may omit variables which influence ridership.

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[5] The methodology used in this research was first proposed in Michael Kyte et al., *Development of Time-Series Based Transit Models*. The reader is urged to refer to that document for an in-depth dissertation on this subject.

Fare elasticity, which measures the responsiveness of ridership to changes in fare, is defined as the percentage change in transit trips resulting from a one percent change in fare, holding constant the effects of all other determining variables. By convention, the Greek letter ε (epsilon) is used here as the symbol for elasticity. The general equation for calculating fare elasticity at any point[6] on the demand function is:

$$\textit{Fare point elasticity} = \varepsilon_{pt} = \frac{\partial R}{\partial FC} \cdot \frac{FC}{R} = \frac{\Delta R/R}{\Delta FC/FC} = \frac{\Delta R}{\Delta FC} \cdot \frac{FC}{R} = \beta \frac{FC}{R} \qquad (3)$$

Notice that after estimating the coefficients (β) of the transit demand function, one may then calculate estimates of elasticity for each of the explanatory variables.

Implicit in the fare elasticity calculation is the fact that its value is dependent upon both the shape of the demand curve and the point on the demand function at which the elasticity is measured. For example, with a linear demand curve elasticity will vary with the values of FC and R and, for comparison purposes, elasticity is usually calculated at the point of the mean of the independent variable. With a hyperbolic demand curve, the elasticity will be constant and the regression coefficient (β) will also be the elasticity value. Also, for forecasting purposes elasticities are more valuable if they are evaluated at the most recent points or over a range of relevant values.

The general demand function shown in equation 1 has been used to model transit ridership and to estimate fare elasticity with varying results. Usually the estimated fare elasticities from such models have been slightly less elastic than the frequently used Simpson-Curtin[7] formula of -0.33. However, serious statistical problems may develop, causing the estimated elasticities to be much different than the true elasticities, when applying the model described in equation 2. In order of importance, these difficulties are outlined below.

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[6] Note that at the limit, where ΔR approaches 0, $\Delta R/\Delta FC = \partial R/\partial FC$

[7] See John F. Curtin, "Effect of Fares on Transit Riding," *Highway Research Record*, 213 (1968), 8-19.

1. **Specification errors.** This type of error includes the omission of major explanatory variables, as well as the inclusion of irrelevant variables and linear estimation of a model which is actually non-linear. These errors will give rise to biased and inconsistent parameter estimates and thereby decrease the confidence one can place in the resultant elasticity estimates. For instance, studies[8] have shown that the average walking time to the bus stop is a significant ridership determinant. However walking time data on a system-wide basis are frequently unavailable and therefore omitted from most estimation models.

Absence of **lagged relationships** is another type of specification error whereby the non-linearity of a function is not explicitly accounted for in the model. An accepted theory of marketing is that the consumer does not respond instantaneously to the introduction of a new product.[9] With regards to mass transit, this theory is plausible, as it will take some time for passengers to learn about a new bus route and to adjust their transportation routines to take advantage of the new service. The true ridership pattern could take six months or longer to fully develop. Some studies recognize this effect by including a lag structure in the model, but most transit analyses have ignored this delayed market reaction.

2. **Multicollinearity.** When a high degree of correlation exists among the explanatory variables, the interpretation of the individual coefficients will be quite difficult. Although there is no loss in the predictive power of the model, the reliability of the individual elasticity estimates will be diminished. Multicollinearity is widespread in business data because economic indicators tend to move in harmony. In many business applications multicollinearity is overlooked, since the general forecasting ability of the

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[8] For example, see Jason C. Yu and Upmanu Lall, *A Bi-Level Optimization Model for Integrating Fare and Service Structures to Minimize Urban Transit Operating Deficits*, pp. 35-46.

[9] See E. Jerome McCarthy, *Basic Marketing*, pp.307-334 for an explanation of the Product Life Cycle Theory.

simulation is usually not affected. Multicollinearity may become a problem when the model is used to determine the influence of a particular factor, such as fares, on the market structure.

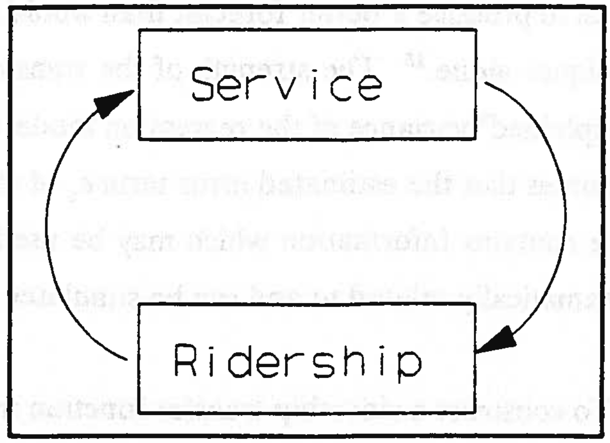
3. Serial correlation of errors. Serial correlation or autocorrelation occurs when the error term of a preceding period is highly correlated with that of a succeeding period. Serial correlation may be sequential, cyclical or some combination of the two. When serial correlation is present, the error terms are not randomly distributed and therefore, the expected value of the sum of the errors is not zero. Violation of this assumption usually has serious consequences. For example, if one is predicting the growth of ridership, an overestimate in one month will lead to an overestimate in succeeding months if serial correlation is present. Also, a serious problem can arise when serial correlation biases the standard error of the regression and leads to the acceptance of a parameter when in reality it should be rejected. Again, the elasticity estimates will be affected. In one case, a model re-estimation with corrections for serial correlation yielded elasticity estimates which were half the original estimates.[10] The Durbin-Watson statistic can be used to test for autocorrelation and corrective techniques, which are available within the ordinary least squares framework, may then be applied.[11] However, the Durbin-Watson test is frequently inconclusive and therefore the effects of serial correlation are sometimes ignored.

4. Failure to account for supply-demand dynamics (simultaneous equations bias). Given a truly simultaneous system, a model which fails to account for the interaction of a dependent variable on an explanatory variable (e.g. ridership on service level) will

[10] Kyte, 1, 57.

[11] The most familiar technique is the Cochrane-Orcutt procedure, but others are available. See Robert S. Pindyck and Daniel L. Rubinfeld, *Econometric Models & Economic Forecasts*, pp.154-158.

Figure 5. Supply-Demand Dynamics



produce estimators which are both inconsistent and biased.[12] Knudson and Kemp[13] partially resolved this problem by designing an instrumental variable for the number of bus miles traveled each month as a function of a set of exogenous variables. The predicted bus miles was then substituted as an instrument for the observed bus miles in the estimation of the demand equations. However, to fully model ridership-service dynamics, a true simultaneous *estimation* model is required, since there are secondary and tertiary effects. As shown in Figure 5, service affects ridership which will affect service which will affect ridership and so on. For example, when ridership declines service is usually reduced, which in turn leads to an additional ridership decrease which may cause further service cuts. Other researchers have worked around this problem by using monthly data since the degree of simultaneity of monthly transit data is certainly less than that of annual data,[14] due to the delay between rider demand and the reaction of the transit agency in increasing or decreasing service. A reasonable assumption is that supply-demand interactions may be overlooked when monthly data are used.

The Transfer Function Model

This model, newly developed in the past decade, combines both time-series analysis and regression analysis to help alleviate the statistical problems previously discussed. It has been

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[12] Ibid., p.339.

[13] Bill Knudson and Michael A. Kemp, *The Effects of a 1976 Bus Fare Increase in the Kentucky Suburbs of Cincinnati*, pp.17-18.

[14] George H.K. Wang and David Skinner, "The Impact of Fare and Gasoline Price Changes on Monthly Transit Ridership: Empirical Evidence From Seven U.S. Transit Authorities," *Transportation Research*, 18B, 29-41.

proven to produce a better forecast than would have been obtained by using either of these techniques alone.[15] The strength of the transfer function model is that it can explain the "unexplained" variance of the regression model which can not be accounted for structurally. It assumes that the estimated error term e_t of the regression model is not truly random and hence contains information which may be used to further define the model and that e_t is mathematically related to and can be simulated by its past values and random disturbances.

To construct a ridership transfer function model, a regression model in which ridership is a function of transit system operating characteristics and local market demographics is built. A time-series model is then formulated to explain the behavior of the residual term of the regression. Lastly, the two model specifications are joined and the coefficients are then estimated simultaneously using non-linear least squares. This technique is relatively new and has proven to be very useful for business forecasting. Although a comprehensive exegesis of the transfer function model is beyond the scope of this paper, the basic methodology as it applies to transit will be covered in this chapter. Readers unfamiliar with the notation and terms used here should consult an econometric or statistical textbook[16] and advanced readers may wish to skip this section entirely.

In order to understand the transfer function model, one must first comprehend the ARIMA (autoregressive, integrated, moving average) model and three time-series analysis concepts: the autoregressive process, the moving average process and stationarity. Auto-regression expounds on the idea that one can forecast the future values of a particular series by examining only the past and current observations of that same series. For mass transit demand, a typical autoregressive equation may take the form:

$$R_t = \phi R_{t-1} + a_t \tag{4}$$

[15] Pindyck and Rubinfeld, p.593.

[16] Suggestions are Henry J. Cassidy, *Using Econometrics: A Beginner's Guide*, Robert Pindyck and Daniel L. Rubinfeld, *Econometric Models & Economic Forecasts*, and Walter Vandaele, *Applied Time Series and Box-Jenkins Models*.

If R_t is June's total ridership, then equation 4 is stating that June's ridership is a ϕ proportion of May's ridership plus an error term a_t, which is random and therefore does not depend on past observations.[17] Higher order autoregressive processes are also possible; equation 5 presents a second order autoregressive model.

$$R_t = \phi_1 R_{t-1} + \phi_2 R_{t-2} + a_t \qquad (5)$$

Following the previous example, equation 5 says that June's ridership is a ϕ_1 proportion of May's ridership plus a ϕ_2 proportion of April's ridership plus an unrelated error. Equation 4 is designated as an AR(1) process while equation 5 is an AR(2) process. The general AR(p) ridership model is given in equation 6 where p is the order of the autoregressive process and corresponds to the number of parameters, ϕ_1 to ϕ_p, which must be estimated.[18]

$$R_t = \phi_1 R_{t-1} + \phi_2 R_{t-2} + \ldots + \phi_p R_{t-p} + a_t \qquad (6)$$

Where the autoregressive process relates R_t to its past values, the moving average process relates the current value of R_t to the present and past random errors, $a_t, a_{t-1}, a_{t-2}, \ldots a_{t-p}$. Using the mnemonics of the autoregressive process, a first order moving average model, MA(1), in which ridership is a function of the current and previous errors is given by:

$$R_t = a_t - \theta_1 a_{t-1} \qquad (7)$$

Equation 7 states that an estimate of ridership at time *t* can be calculated by multiplying the past error, a_{t-1} by the moving average parameter, θ_1 and then subtracting this from the current error, a_t. Of course higher order moving average models are also possible and the general moving average model is specified as MA(q), where q is the order of the moving average process and corresponds both to the number of parameters (θ_1 to θ_p) which need

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[17] Notice the use of a_t which is a truly random or "white noise" process instead of e_t which may not be white noise due to the statistical problems explained previously.

[18] Walter Vandaele, *Applied Time Series and Box-Jenkins Models*, p.39.

to be estimated and the number of lagged a_t's. The form of the qth order moving average process, MA(q), is represented by equation 8.

$$R_t = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q} \quad (8)$$

As with most statistical forecasting techniques, time-series modeling requires that certain statistical relationships are assumed. The most important assumption is that the time-series data must be **stationary**. To be stationary, the following three conditions must hold true for a particular time-series:

1. the mean is constant so that the mean of $R_t = E(R_t) = \mu$

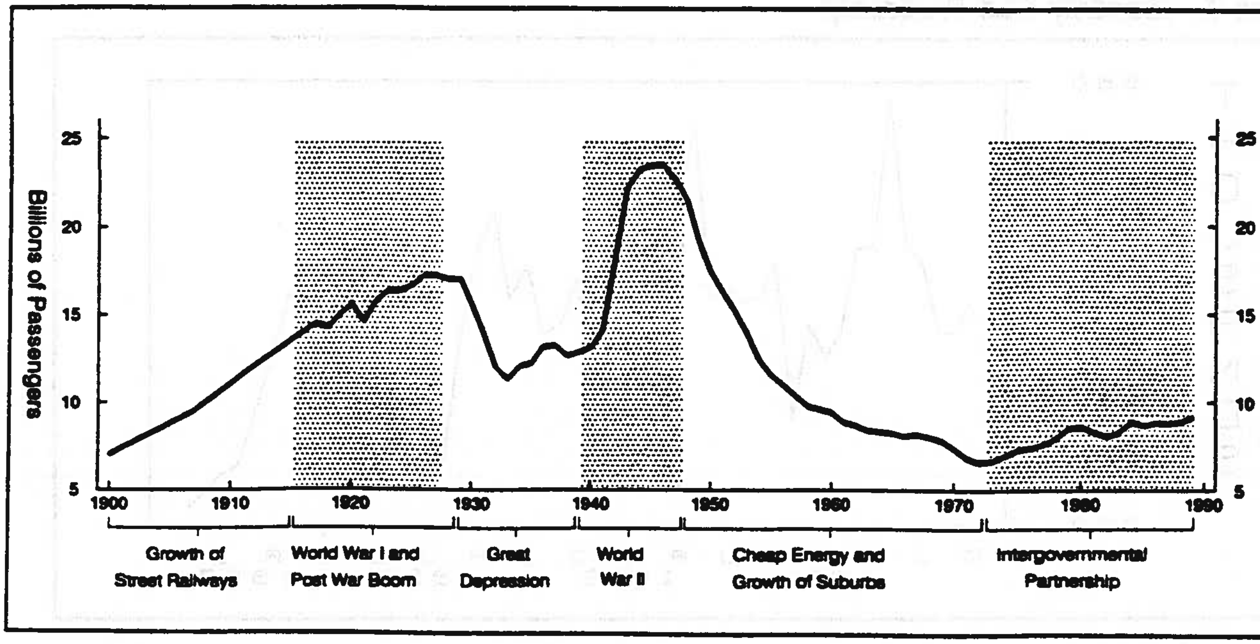
2. the variance is constant so that the variance of $R_t = E[(R_t - \mu)^2] = \sigma^2$

3. the correlation between a series value at time t (R_t) and a series value at time t-k (R_{t-k}) depends only on the time lag k and not on the time period, t. Mathematically, this may be represented as:
the autocorrelation $(R_t, R_s) = E[(R_t - \mu)(R_s - \mu)]/\sigma^2 = \rho_{t-s}$

The reason for the stationarity assumption may be obvious. Time-series analysis attempts to simulate the underlying stochastic process which generated the original data. It is only if the characteristics of this underlying process are invariant with respect to time can one represent the time series over past and future intervals by a simple algebraic model.[19] That is, only if the stochastic process is stationary and therefore fixed in time, can one model the process by an equation with fixed coefficients that can be estimated from past data. Those readers who are familiar with regression analysis may recall similar assumptions whereby the structural relationships between the dependent variable and the explanatory variables are assumed to have remained constant over time and about the homoscedasticity of the error terms.

[19] Pindyck and Rubinfeld, p.497.

Figure 6. Major Trends of Transit Ridership



In reality, most time series used in business analysis are not stationary and display some sort of consecutive or cyclical trend, as is true with transit ridership. Figure 6, Major Trends of Transit Ridership,[20] shows the trend of U.S. transit ridership from 1900 to 1989. Both the mean and variance of annual ridership are not constant over time. Of course, monthly data only exacerbates the non-stationarity of the existing ridership trends by introducing seasonal cycles as can be seen in Figure 7, Monthly Bus Ridership.[21] Therefore, all three time-series assumptions have been violated with the result that transit ridership data is non-stationary.

The first step then, is to transform transit ridership from a non-stationary series to a stationary series before time-series modeling techniques can be employed. To make a data series stationary, one must first reduce the inherent volatility, and so induce a constant variance, and second, remove any consecutive or cyclical trends. Possible mathematical transformations to stabilize the variance include logarithmic, square root and power

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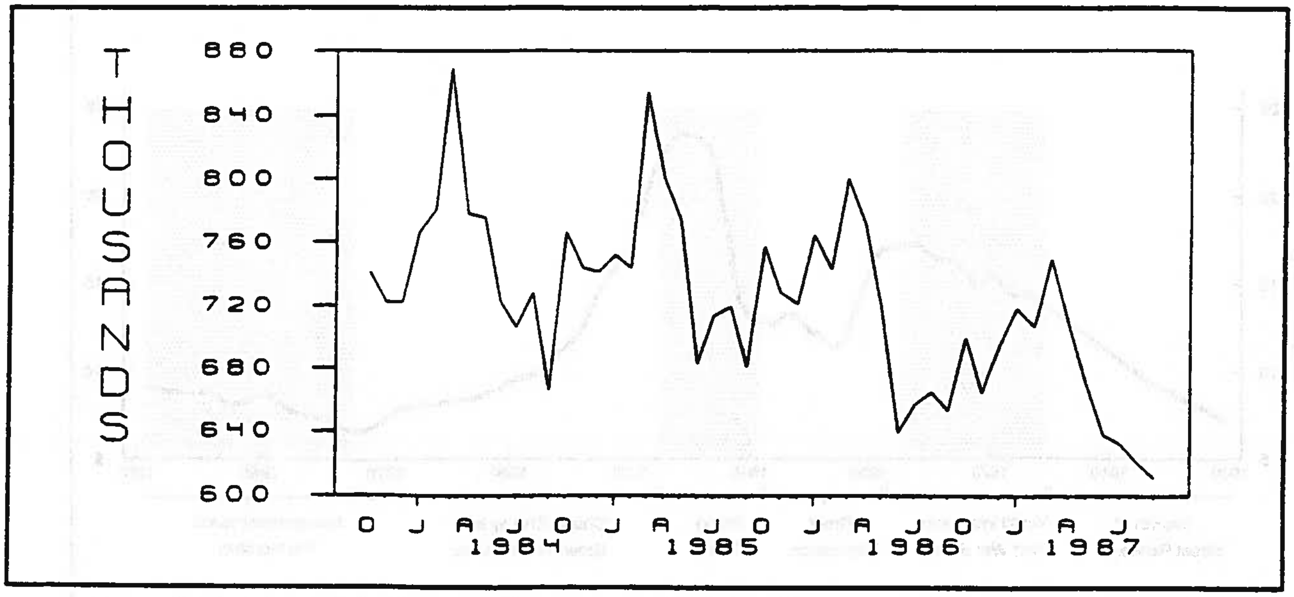
[20] American Public Transit Association, *1990 Transit Fact Book*, p.40.

[21] The data for Figure 7 is from the Pinellas Suncoast Transit Authority, Clearwater, Florida.

Figure 7. Monthly Bus Ridership



functions. The log transformation is especially effective when the variance of the series is proportional to the mean and when the mean changes at a constant percentage rate,[22] which is usually the case with business data. Transformations to remove the trend include consecutive differencing and also seasonal differencing for monthly or quarterly data. For monthly transit ridership, it has been found that natural log[23] transformations with first order consecutive differencing and first and maybe second order (12 and 24 month) seasonal differencing are usually sufficient to generate a stationary series. In practice the logarithmic transformation must be performed first because differencing will produce negative values for which the log is undefined. These transformations, which are necessary to achieve stationarity, make up the *integrated* part of the ARIMA acronym.

Having separately illustrated both the autoregressive and moving average processes and the concept of stationarity, the next step is to merge them into the general ARIMA(p,d,q) model, which is defined in equation 9.

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[22] Vandaele, p.18.

[23] By convention natural logarithm (log to the base e) is used in this report.

$$\phi(B)(1-B)^d \ln(R_t) = \theta(B)a_t \tag{9}$$

where:

- B = the backward shift (or backshift) operator where: $B(R_t)=R_{t-1}$ and $B^k(R_t)=R_{t-k}$
- ϕ = the autoregressive characteristic polynomial
- θ = the moving average characteristic polynomial
- d = the order of differencing required to induce stationarity in the series R_t
- p = the order of the autoregressive process
- q = the order of the moving average process
- a = the random error term
- R = the transit ridership time-series
- ln = the natural logarithm operator

In the context of transit, the ARIMA(0,1,1) model, which has no autoregressive process, says that the first difference, $R_t - R_{t-1}$ satisfies the MA(1) model. Mathematically, this can be expressed as:

$$R_t - R_{t-1} = a_t - \theta_1 a_{t-1} \tag{10}$$

or

$$R_t = R_{t-1} + a_t - \theta_1 a_{t-1} \tag{11}$$

The ARIMA(1,1,0) model, which has no moving average process, states that the first difference, $R_t - R_{t-1}$ satisfies the AR(1) model. Mathematically, this is expressed as:

$$R_t - R_{t-1} = \phi(R_{t-1} - R_{t-2}) + a_t \tag{12}$$

or

$$R_t = (1 + \phi)R_{t-1} - \phi R_{t-2} + a_t \tag{13}$$

The usefulness of the ridership model would be greatly enhanced if it could represent transit's seasonal nature. The model should be able to exploit correlation between the current month's ridership level and the ridership level of the same month in the previous year(s). In fact, another form of the ARIMA model, called the general multiplicative seasonal model, can account for seasonality. This model is symbolized as

ARIMA(p,d,q) × $(P,D,Q)_s$ where P is the order of the seasonal autoregressive process, D is the order of seasonal differencing, Q is the order of the seasonal moving average process and s is the span of seasonality.[24] Mathematically, this model is expressed as:

$$\phi(B)\Phi(B)(1-B)^d(1-B^s)^D \ln(R_t) = \theta(B)\Theta(B)a_t \tag{14}$$

where:

- d = the order of non-seasonal differencing required to induce stationarity
- D = the order of seasonal differencing required to induce stationarity
- ϕ = the non-seasonal autoregressive characteristic polynomial of the form: $\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots - \phi_p B^p$
- Φ = the seasonal autoregressive characteristic polynomial of the form: $\Phi(B) = 1 - \Phi_1 B^s - \Phi_2 B^{2s} - \ldots - \Phi_P B^{Ps}$
- θ = the non-seasonal moving average characteristic polynomial of the form: $\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q$
- Θ = the seasonal moving average characteristic polynomial of the form: $\Theta(B) = 1 - \Theta_1 B^s - \Theta_2 B^{2s} - \ldots - \Theta_Q B^{Qs}$
- s = the span of seasonality (i.e. s=12 months, 2s=24 months for monthly data)
- B = the backshift operator where $B(R_t)=R_{t-1}$ and $B^k(R_t)=R_{t-k}$
- a = the random error term
- R = the transit ridership time-series
- ln = the natural logarithm operator

As with the non-seasonal ARIMA model, perhaps an example[25] may serve to clarify the methodology. Consider the case of the first order consecutive and seasonal autoregressive model, $ARIMA(1,0,0)\times(1,0,0)_{12}$. Suppose that monthly ridership R_t can be represented by the first-order seasonal AR model or SAR(1) model:

$$R_t - \Phi_1 R_{t-12} = e_t \tag{15}$$

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[24] Vandaele, p.58.

[25] Ibid., pp.59-60.

where Φ_1 is the seasonal first-order AR parameter and alternatively can be written using the backshift operator (B) as:

$$(1 - \Phi_1 B^{12})R_t = e_t \tag{16}$$

Assuming that the error term e_t is not purely random, it could be represented by a first-order autoregressive process in the consecutive months, AR(1), as shown in equation 17.

$$e_t = \phi_1 e_{t-1} + a_t \tag{17}$$

or, again using the backshift operator:

$$(1 - \phi_1 B)e_t = a_t \tag{18}$$

Here a_t represents the purely random white noise process. Equation 18 can be thought of as representing the influence of the successive trend on transit ridership while equation 16 can be thought of as representing the effect of the seasonal trend on ridership. Replacing e_t in equation 18 with equation 16 gives the first-order consecutive and seasonal autoregressive model ARIMA(1,0,0)×(1,0,0)$_{12}$ for transit ridership:

$$(1 - \phi_1 B)(1 - \Phi_1 B^{12})R_t = a_t \tag{19}$$

A similar equation can be developed for the consecutive and seasonal moving average model and the two models may then be combined. In practice, the orders of the parameters p, d, q, P, D and Q are all small, typically two or less[26] and are determined through analysis of the autocorrelation function and the partial autocorrelation function of the residuals. The reader is urged to refer to an econometric text for in depth instruction in the use of these tests for specification of an ARIMA model.

[26] Kyte, 2, 32-33.

Model Application: Estimating the Transit Demand Function

The last step in building a transfer function model is to combine the structural regression[27] model with the ARIMA model. Many forms have been proposed for the structural model, but unlike the time-series model which can be estimated using only past ridership data, the regression model is constrained by the availability of explanatory data. When a critical explanatory variable's data are unobtainable, a suitable proxy must be used. All of the transit proxies used in this study were chosen because they are standardized statistics, reported to the Urban Mass Transportation Administration by transit systems receiving federal funds under Section 9 of the Urban Mass Transportation Act of 1964 as Amended, and hence should be kept by most urban public transit agencies.

The Variables

As a measure of transit demand, system-wide unlinked passenger trips were used as a proxy for system-wide ridership, R. Unlinked passenger trips are:

> Transit trips taken by both initial-board (originating) and transfer (continuing) transit patrons. Each passenger is counted each time that person boards a transit vehicle regardless of the type of fare paid or transfer presented.[28]

Using unlinked passenger trips as a ridership proxy raises the bus transfer problem. For example, a patron may take two different buses to work, but perceives the two bus rides as only one "work trip." Linked trips probably better represent how a bus rider sees transit service, but unfortunately linked trips are not commonly reported. Hence, unlinked trips were used with the assumption that the ratio of transfers to unlinked trips remained constant over the observation period.

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[27] Unlike time-series analysis, the basics of regression analysis will not be covered here as many transit planners are already familiar with this technique.

[28] American Public Transit Association, *1987 Transit Fact Book*, p.80.

Level of service and accessibility supplied by the transit system SL, is the major determinant of transit ridership. For level of service, either vehicle revenue miles or vehicle revenue hours were used as a proxy.[29] A probable drawback of this substitution is that these surrogates are gross measures of system service and may not capture important service factors such as reliability, in-vehicle time, seat availability, waiting time and walking time to the bus stop. However, vehicle miles or vehicle hours will capture significant route extensions or contractions which affect waiting time and in-vehicle time. Dividing vehicle revenue miles by vehicle revenue hours yields a measure of average bus speed and changes in bus speed roughly approximate in-vehicle time changes. Assuming that walking distances, seat availability and reliability do not drastically change over the observation period, vehicle revenue miles and revenue vehicle hours should be acceptable measures of service level.

Intuitively, the coefficient of SL should be positive; as the level of service increases, ridership should also increase. An aggregation[30] of 23 cases examined in previous studies produced a mean bus vehicle miles elasticity of +0.64 ± 0.30. One would also suspect that a lag relationship exists between ridership and service levels. A study by Kyte et al. suggests that "the impacts of service level changes in the urban sectors lagged about two quarters [when analyzing quarterly data] or eight to ten months [when analyzing monthly data]. Suburban service changes usually showed less of a delay, often one quarter or one month."[31] Accordingly, in formulating the explanatory structure, service level lags of one to twelve months were tested by the cross correlation function along with the current month's service level.

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[29] The definition of Vehicle Revenue Miles is "total number of miles traveled by revenue vehicles while in revenue service. Excludes miles traveled to and from storage facilities and other deadhead travel." The definition of Vehicle Revenue Hours is "total number of hours that a vehicle is in revenue service. Excludes hours consumed while traveling to and from storage facilities and during other deadhead travel." These definitions are from *National Urban Mass Transportation Statistics, 1983 Section 15 Annual Report*, Urban Mass Transportation Administration, p.C-4.

[30] Patrick Mayworm, et al., *Patronage Impacts of Changes in Transit Fares and Service*, p.65.

[31] Kyte, 1, 57.

For the total cost of traveling by mass transit FC, two fare measures were tested in the model, the base adult cash fare (base fare) and the average fare. Both fare measures were deflated by the appropriate CPI so that the real fare changes were measured in constant instead of nominal dollars. The definition of the base adult cash fare in this report is:

> The amount of fare paid for a single ride, excluding zone and transfer charges, during the off-peak period by passengers who are not entitled to reduced fares and who pay the fare with money.

There are a few problems with the base fare. First, it is not applicable to a significant number of passengers, such as express and discount riders. Second, when the base fare is increased, the percentage change is not usually the same for all riders, especially if the fare structure has been altered contemporaneously. On the positive side, the base fare is easily incorporated into the model and any change in the base fare unequivocally marks some sort of price change for most passengers.

The average fare, which is calculated by dividing passenger fare revenues for transit service[32] by unlinked passenger trips, is not without criticism. First, unlike the base fare, the average fare is subject to measurement errors. Although the errors are expected to be smallest in those systems which have registering fare boxes in their vehicles, an estimate of the size and effects of these errors is not available. Second, the average fare will change depending on ridership composition. For instance, student discounts, which are not frequently used in the summer, will increase the summer months' average fare even though the real price facing the transit rider has not changed at all. More importantly, the average fare is usually not the price that the consumer at the margin, upon whom the pivotal economic concept of marginal utility is based, must pay. Ideally, one would create a

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[32] The definition of passenger fare revenues for transit service is "revenues earned from carrying passengers along regularly scheduled routes. Includes the base fare, zone premiums, extra cost transfers and quantity purchase discounts applicable to the passenger's ride. Also included is 'park and ride' revenue. Excluded are charter service revenues, school bus service revenues, non-transportation revenues and all fare revenue retained by the contractor operating the route."

weighted average fare using the number of passengers who paid that fare as the weight for each fare category. This weighted average fare would then be deflated by the local CPI. However, it was deemed that reporting data by fare category would be burdensome to the survey respondents.

The sign of the coefficient for FC should be negative. Economic theory states that as the price of a good or service increases, less of that good or service will be consumed. An aggregation[33] of 12 cases in prior studies produced a mean bus fare elasticity of -0.35 ± 0.14. Note that this value is similar to the Simpson-Curtin formula.

Intuitively, a fare increase would cause a wide range of responses over time. Some patrons who could easily switch to an alternate mode, like walking, would leave the system immediately. Other patrons would consider alternate transportation and begin taking action, such as buying an auto or making carpool arrangements, which would empower them to modify their transit routines in the near future. Still other patrons would make long-run adjustments such as finding employment nearer to their home or moving closer to their workplace. Kyte et al., found that the "*effects of fare changes typically were instantaneous, usually occurring in the same period as the implementation of the fare change. However, some effects were measured for one month after a fare change for the system data thus suggesting an exponential decay function.*"[34] Other authors have speculated that the sharp ridership decline experienced right after a fare increase is only a drastic, short-lived rebuttal of the price increase, akin to a consumer boycott. This line of thought goes on to explain that a good portion of the passengers lost during the first few weeks after a fare increase will be recouped in later months.[35] Another researcher, Kemp, states that "*most of the ridership*

[33] Mayworm, p.x.

[34] Kyte, 1, 58.

[35] Transport and Road Research Labs, *The Demand for Public Transport*, p.112.

response to a fare change is experienced over about six to nine months following the change."[36] In any case, it seems that a lag structure is vital to understanding the transit market's reaction to price changes. Therefore, like the service variable, the cross correlations between ridership and the current value and first 12 lag values of fare were examined.

There are other costs incurred by transit patrons. Some of these costs, such as lack of privacy and comfort, have an enormous range of personal utilities. Others, like waiting time and walking time, are very difficult to quantify on a system-wide basis. Because of measurement problems, these costs have not been included. This is not to deny their existence. It is merely that a structural model of their effects could not be built within the framework and resources of this study.

For the variable AC, the total costs of traveling by an alternate or competing mode, the local gasoline price deflated by the local CPI was used. Although there are other alternate modes, such as walking, bicycling and rail transit, whose costs are not related to gasoline prices, the automobile is by far transit's predominant competitor. Furthermore, it was assumed that only variable costs are relevant in comparing transit costs with auto costs. The justification is that the consumer does not consider fixed auto costs, such as depreciation and insurance, in the short-term modal travel decision.

The primary automobile variable costs are gasoline and oil at 5.86 cents per mile, maintenance at 1.12 cents per mile and tires at 0.64 cents per mile.[37] Of these three costs, gasoline is the largest and the only daily out-of-pocket expense. These figures suggest that auto drivers would consider mostly gasoline costs and therefore gasoline price should be the primary factor in the modal travel decision. Other large out-of-pocket automobile costs are

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[36] Michael A. Kemp, *Planning For Fare Changes: A Guide To Interpreting and Using Fare Elasticity Information For Transit Planners*, p.36.

[37] American Automobile Association, *Your Driving Costs*, 1980 edition, p.3.

parking and tolls. A study[38] in the Twin Cities area shows that the cross elasticity of bus demand to automobile parking costs for work trips in the CBD is +0.51. Clearly, parking costs are very important. However, these costs are also extremely geographically diverse, even within large cities, and vary from a high of $300 per month in Manhattan to a low of $50 per month in Miami.[39] Consequently, no readily available national time-series data exists and they are excluded from the model.

Concerning prior research, an aggregation[40] of four cases in earlier studies produced a bus demand to auto operating costs cross elasticity of +0.74 ± 0.23. This is theoretically consistent since the cross elasticity between a good or service and its substitute should always be positive. That is, as the price of the good or service increases, the demand for the substitute should also increase and vice versa. There are two cases of interest here. One case is where the car driver leaves his auto for the bus and the second case is where the bus passenger abandons the bus for a car. In the first case, the ridership impact should occur shortly after a significant automobile operating cost increase, since the skills and resources needed to patronize mass transit are easily acquired. In the second case, one could envision a longer lag since the bus-to-auto switch may involve the purchase of equipment or the obtaining of a driver's license.

Local employment, as reported by the U.S. Bureau of Labor Statistics in the monthly periodical *Employment and Earnings*, was used as a surrogate for the travel market characteristics, MC. Cross sectional studies frequently use additional demographic data such as population income and age as measures of the size of the travel market. This data, collected by the Census Bureau, is not readily available on a monthly basis. The local

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[38] Mayworm, p.29.

[39] David Landis, "Prices For Big-City Parking Places Roll Higher," *USA Today*, September 10, 1987, p.1.

[40] Mayworm, p.28.

employment figure is an excellent measure of the work trip travel market, but a poor measure of the market size for some other types of transit rides, such as trips taken by the elderly, students and the very poor. However, remember that employment is closely related to variables such as income and the economic level in general, which have a powerful influence on all segments of the transit market. Some evidence of this can be gained by referring to Figure 6, Major Trends of Transit Ridership, where one can see that the Great Depression caused a steep decline in ridership between 1929 and 1939. Consequently local employment should be a good proxy for travel market size in a system-wide demand study.

Since demand for mass transit is dependent upon the level of travel inducing activities,[41] ridership should be strongly and positively related to the general economic level. Therefore, as employment decreases (increases) then ridership should also decrease (increase). Previous studies have shown that employment elasticities range from +0.50 to +0.70.[42] One researcher reported an employment elasticity of 1.086, meaning that a 1.086 percent increase in ridership occurs for each one percent increase in jobs.[43] Conceivably, employment elasticities greater than one could occur when employment is increasing and when the newer workers have different transportation practices than the existing workers. There is some evidence that this happened during the time frame of this study when a significant number of women joined the workforce. From 1980 to 1987 the U.S. civilian labor force increased by 13 million, of which 63 percent were female.[44] Also, 12.1 percent of female workers used public transportation for the worktrip, while only 7.5 percent of male

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[41] Kemp, *Planning For Fare Changes*, p.8.

[42] Kyte, 1, 58.

[43] D.C. Agrawal, "The Factors Affecting Mass Transportation Ridership: An Analysis," *Transit Journal*, Summer 1987, p.64.

[44] U.S. Department of Commerce, Bureau of the Census, *Statistical Abstract of the United States 1989*, p.376

workers used public transportation.[45] Lastly, since 51 percent of U.S. households have an average of less than two vehicles,[46] the second working adult may have had to use mass transit. Therefore, it is probable that the new workers used mass transit at a higher rate than the existing workers. An example may be conducive to understanding this concept. Suppose that in a hypothetical city there are 1,000 workers and five percent of these workers, or 50 workers, use mass transit to and from work for a total of 100 unlinked trips daily. Now, consider the case where 100 new people join the work force and of the 100 new workers, six percent use mass transit to and from the job. The new employment figure is 1,100 and the new ridership figure is 112 [100 + (6% × 100 workers × 2 trips daily)]. The employment elasticity of demand (ε_E) as calculated by the midpoint arc elasticity formula is shown below to be 1.19.

$$\varepsilon_E = \frac{(R_2-R_1)}{(R_2+R_1)/2} \div \frac{(E_2-E_1)}{(E_2+E_1)/2}$$

$$\varepsilon_E = \frac{(112-100)}{(112+100)/2} \div \frac{(1100-1000)}{(1100+1000)/2} \qquad (20)$$

$$\varepsilon_E = 1.19$$

In other words, an employment elasticity greater than one implies that the newly employed workers utilize mass transit at an **only slightly higher rate** than the existing workers, which is reasonable for the 1980s.

Although the effects of reduced employment should impact transit ridership immediately and therefore no lag structure is expected, employment increases may exhibit a lagged effect on ridership. The reason for this is that people entering the workforce may not use mass transit until they become accustomed to their new working schedule and environment. Also,

[45] William O'Hare and Milton Morris, *Demographic Change and Recent Worktrip Travel Trends*, I, A1.

[46] Motor Vehicle Manufacturers Association of the United States, Inc., *MVMA Motor Vehicle Facts & Figures '89*, p.45.

increased employment usually exhibits a slow trickle-down effect on the general economy and hence on activities, such as shopping, which motivate travel. In brief, with monthly data no lagged relationship between ridership and decreasing employment is expected, although the lag cross correlations should be examined when employment is increasing.

Other urban characteristics which affect the transit market are city age, size and dimensions, urban characteristics and geographic location. Again, these factors should not change significantly in the short-run and are irrelevant for this fare elasticity analysis.

As stated previously, transit ridership traditionally exhibits strong seasonal variation. There are three basic causes of this variation: 1) adverse winter weather, 2) summer vacations, and 3) the number of working days in the month. This seasonality can be handled two ways within the framework of the transfer function model. First, one may choose to structurally model the seasonal characteristics by using month length variables, seasonal dummy variables and weather variables.[47] The major drawback to this method is that the seasonal variables are geographically diverse. For instance, it would be very time consuming, but certainly not impossible, to obtain and input all available monthly temperature and precipitation data for the United States. The second method is to use the ARIMA general multiplicative seasonal model and simply add a seasonal multiplicative component to the error structure of the transfer function model.[48] The approach taken here is to attempt to model the monthly working day variation along with the other structural parameters and to also use the seasonal ARIMA model to capture the remaining seasonal effects. Obviously, the number of working days per month is readily available for both past and future periods. According to Wang and Skinner, "the rationale for the inclusion of working-day variation in the model ... [is that the] ... autocorrelation functions of residuals fail to detect working-day variation omitted from the series and the inclusion

[47] Kemp, *Planning For Fare Changes*, p.33.

[48] Kyte, et al., 1, 26.

of working-day variables in the model provides us with a direct test on the existence of working-day variation in the series as well as a procedure for directly estimating it from the data."[49] The disadvantage to incorporating working days into the equation is that it is highly correlated with the service variables and hence its inclusion may lead to multicollinearity problems.

The last variable which may be added to the model is the intervention variable, also known as the structural dummy variable. Interventions are discrete events like strikes, fuel shortages, special promotional activities, and new laws which have a measurable impact upon ridership. Two characteristics of the intervention must be specified *a priori*, its starting point and the general shape or expected nature of the impact of the intervention. For some interventions, such as strikes, the starting point and its effect upon ridership are easily ascertained. Conversely, it may be difficult to determine exactly when a fuel shortage started, although the general shape of its impact on ridership may be evident from the data.

The intervention variable can take a value of either 0 or 1 at any given observation point. For example, suppose that a new law is enacted which would make all major arterial highways HOV-4 (high occupancy vehicles, 4 persons) during rush hours. Many commuters who previously drove to work on these highways would now have to make arrangements to travel in a vehicle which was carrying four or more persons. Transit ridership and carpools should increase. In this case the intervention variable I_t would take the value of 0 prior to the law's execution and 1 thereafter. This is called a step function, because the intervention has a permanent *step* effect upon the level of ridership. Another case involves interventions of fixed length which do not have lasting effects. Here I_t takes the value of 1 during the event and 0 otherwise. For instance, I_t would be 1 during a strike and 0 before and after the strike. This is referred to as a pulse function.

[49] Wang and Skinner, p.33.

The shape of the intervention can usually be classified among one of the following general forms:[50]

1. Abrupt start and the effect of the intervention is of permanent duration.
2. Gradual start and the effect of the intervention is of permanent duration.
3. Abrupt start and the effect of the intervention is of temporary duration.
4. Gradual start and the effect of the intervention is of temporary duration.

Combinations of these four general shapes are also possible. A "no pay" promotion would increase ridership during the free-ride time period (type 3) and would, hopefully, have a positive residual effect after the promotion ended (type 1). Besides being a method for dealing with data perturbations, intervention analysis permits the transit planner to answer questions like "What was the effect of the two month marketing promotion on my permanent ridership?"

Identifying and assembling the appropriate proxies is perhaps the most difficult step in econometric modeling. Table 5 shows the theoretical variables and the corresponding measures selected for this study.

Table 5. Structural Variables and Measures Used in Transit Demand Model

VARIABLE	ACRONYM	MEASURE (PROXY)
Ridership	R	Unlinked Passenger Trips
Service Level	SL	Revenue Vehicle Miles and/or Hours
Transit Cost	FC	Real Average Fare
Alternate Mode Cost	AC	Real Local Price of Gasoline
Market Size	MC	Local Employment

[50] Vandaele, p.336.

The Demand Function

Having worked through the mechanics, the actual combination of time-series and regression analysis methods can now be performed. Restating equation 2 in a form which allows for lag structures, replacing the general seasonal variable S_t with the working day variable WD_t and temporarily omitting the intervention variable I_t gives equation 21.

$$\hat{R}_t = \nu_0 + \nu_1'(B)SL_t + \nu_2'(B)FC_t + \nu_3'(B)AC_t + \nu_4'(B)MC_t + \nu_5'(B)WD_t + e_t \quad (21)$$

where:

$$\nu_1'(B)SL_t = \nu_{10}SL_t + \nu_{11}SL_{t-1} + \nu_{12}SL_{t-2} + \ldots \quad (22)$$
$$\nu_2'(B)FC_t = \nu_{20}FC_t + \nu_{21}FC_{t-1} + \nu_{22}FC_{t-2} + \ldots \quad (23)$$
$$\nu_3'(B)AC_t = \nu_{30}AC_t + \nu_{31}AC_{t-1} + \nu_{32}AC_{t-2} + \ldots \quad (24)$$
$$\nu_4'(B)MC_t = \nu_{40}MC_t + \nu_{41}MC_{t-1} + \nu_{42}MC_{t-2} + \ldots \quad (25)$$
$$\nu_5'(B)WD_t = \nu_{50}WD_t + \nu_{51}WD_{t-1} + \nu_{52}WD_{t-2} + \ldots \quad (26)$$

In practice, some of the explanatory variables, such as working days, will not have a lag structure and therefore only the current time period parameter (ν_{50}) will be significant. Other variables, such as service level and fare, are expected to have significant lag structures. A problem then arises when attempting to estimate the lag parameters, $\nu_{11}, \nu_{12}, \nu_{13}, \ldots$ and $\nu_{21}, \nu_{22}, \nu_{23}, \ldots$, since an infinite number of functionally unrelated parameters can not be estimated from a finite set of observations. These parameters must be represented in a parsimonious form whereby the parameters are assumed to be functionally related. The two most common forms are Almon lags, in which the coefficients are related by a polynomial function of prespecified degree and duration, and Koyck lags or decreasing geometric distributed lags, in which an exponential decay function is used.[51] Consider that equation 22 can be rewritten with just two parameters, ω_1 and δ, by using a geometric lag as shown in 27.

[51] Kemp, *Planning For Fare Changes*, p.36.

$$\nu_1'(B)SL_t = \omega_1(SL_t + \delta SL_{t-1} + \delta^2 SL_{t-2} + \delta^3 SL_{t-3} + \ldots) \quad (27)$$

or

$$\nu_1'(B)SL_t = \omega_1 \sum_{s=0}^{\infty} \delta^s SL_{t-s} \quad (28)$$

As shown in equation 28, the *long-run response* in a geometric lag model such as that in equation 27 is simply the parameter ω_1 times the sum of the lag weights $\Sigma\delta^s$ or, more appropriately, $\omega_1/(1-\delta_1)$.[52] Now the ν parameters can be readily represented in a parsimonious form as demonstrated in equation 29.

$$\nu_1'(B)SL_t = \frac{\omega_1}{1-\delta_1 B} SL_t \quad (29)$$

The parameters in the polynomial ω_1 are commonly referred to as the *numerator parameters* and in polynomial δ_1 as the *denominator parameters*.[53] Using numerator and denominator polynomials to represent the lag structures, equation 21 may now be rewritten as:

$$\hat{R}_t = \omega_0 + (\omega_1/(1-\delta_1 B))SL_t + (\omega_2/(1-\delta_2 B))FC_t + \omega_3 AC_t + \omega_4 MC_t + \omega_5 WD_t + e_t \quad (30)$$

Equation 30 can be interpreted as saying that there are five factors which influence transit ridership: service level (SL), fares (FC), alternate modes' costs (AC), market characteristics (MC) and working days per month (WD) plus an error term, e_t. Furthermore, the effects of service level and fare changes begin immediately and decay over the next several periods, while alternate modes' costs, market characteristics and working days affect ridership only in the current period.

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[52] Recall that the sum of an infinite series such as $\Sigma\delta^s = 1/(1-\delta)$. See Pindyck and Rubinfeld, p.232.

[53] Vandaele, p.263.

The last step in the transfer function model is to use time-series analysis to construct an ARIMA model for the error structure e_t as a function of past values and random disturbances. This ARIMA model will then be substituted for the error term of the original regression equation. Again, the reason that the transfer function model is superior to a regression model is that it *explains* the unexplained variance of the regression equation and thereby addresses the problems of regression analysis which were outlined previously. Using the now familiar ARIMA notation, equation 31 proposes the form of this model.

$$\phi(B)\Phi(B)(1-B)^d(1-B^s)^D e_t = \theta(B)\Theta(B)a_t \quad (31)$$

or

$$(1-B)^d(1-B^s)^D e_t = \frac{\theta(B)\Theta(B)}{\phi(B)\Phi(B)} a_t \quad (32)$$

In order to complete the transfer function model, the right side of equation 32 is substituted into equation 30. To reiterate an important point, the ridership variable will usually require function transformations and both consecutive and seasonal differencing to induce stationarity and these same transformations and differencing carry over to the explanatory variables. After these transformations, the model becomes awkward, so the convention of using lower case letters to represent the dependent and explanatory variables after natural log transformation and differencing is introduced here. That is:

$$r_t = (1-B)^d(1-B^s)^D \ln R_t \quad (33)$$

$$sl_t = (1-B)^d(1-B^s)^D \ln SL_t \quad (34)$$

$$fc_t = (1-B)^d(1-B^s)^D \ln FC_t \quad (35)$$

$$ac_t = (1-B)^d(1-B^s)^D \ln AC_t \quad (36)$$

$$mc_t = (1-B)^d(1-B^s)^D \ln MC_t \quad (37)$$

$$wd_t = (1-B)^d(1-B^s)^D \ln WD_t \quad (38)$$

Finally, equation 39 proposes one possible form of a bus ridership transfer function model.

$$\hat{r}_t = \omega_0 + \frac{\omega_1}{1-\delta_1 B} sl_t + \frac{\omega_2}{1-\delta_2 B} fc_t + \omega_3 ac_t + \omega_4 mc_t + \omega_5 wd_t + \frac{\theta(B)\Theta(B)}{\phi(B)\Phi(B)} a_t \quad (39)$$

It is likely that some of the variables will not be statistically significant and it would not be unusual to drop or add variables, numerator parameters (ω) or denominator parameters (δ) from the model. If needed, the intervention variable can easily be added to the model as shown in equation 40, where I is the intervention variable, T is the time period at which the event starts and ψ is the intervention form.

$$\hat{r}_t = \omega_0 + \frac{\omega_1}{1-\delta_1 B} sl_t + \frac{\omega_2}{1-\delta_2 B} fc_t + \omega_3 ac_t + \omega_4 mc_t + \omega_5 wd_t + \frac{\theta(B)\Theta(B)}{\phi(B)\Phi(B)} a_t + \psi I_t^T \qquad (40)$$

Strengths of the Transfer Function Model

A comparison of the transfer function model represented in equation 40 with the regression model represented in equation 2 points out the superiorities of the transfer function in modeling transit ridership. Recall that one of the most vexing dilemmas of structural regression models using business data is multicollinearity. The transfer function solves this problem by relating the *differences* of the explanatory variables to the differences of the dependent variable. Differences usually have a much lower correlation coefficient than the nominal values of the explanatory variables. In fact, regression models sometimes use the percent change of the explanatory variables as one remedy for multicollinearity. Percentage changes are essentially changes in logs which are the same as first order differencing of a log transformation.[54] The drawback to using differenced variables is that the R^2 values are generally higher when nominal values are used.[55] One strength of the transfer function model then, is that it provides a structured framework within which transformations that reduce multicollinearity are performed systematically.

Serial correlation is another regression problem addressed by the transfer function. The residuals of the transfer function's structural component will still be serially correlated, but

[54] Charles R. Nelson, *Applied Time Series Analysis for Managerial Forecasting*, p.58.

[55] Cy Ulberg, "Short-Term Ridership-Projection Model," *Transportation Research Record*, 854, 15.

this serial correlation is then modeled by the ARIMA component. Of course without serial correlation the ARIMA component of the transfer function would be ineffective. The outcome is that serial correlation does not affect the structural model of the transfer function as severely as the uncorrected regression model.[56]

The transfer function also lessens the impact of two types of specification errors which commonly affect regression models, 1) the absence of lagged relationships, and 2) the omission of relevant variables. As shown previously, the transfer function includes a methodology to explicitly investigate lag relationships through the cross correlation function. Secondly, the trend of the explanatory variables which were omitted from the structural regression model may be modeled by the ARIMA specification. For example, assume that the true ridership demand model is given in equation 41.

$$R_t = \beta_0 + \beta_1 SL_t + \beta_2 FC_t + \beta_3 AC_t + \beta_4 MC_t + \beta_5 S_t + \beta_6 I_t + \epsilon_t \qquad (41)$$

Now, suppose that the researcher omits alternate modes' costs (AC_t) and instead estimates equation 42 as the regression model.

$$\hat{R}_t = \beta_0 + \beta_1 SL_t + \beta_2 FC_t + \beta_4 MC_t + \beta_5 S_t + \beta_6 I_t + e_t \qquad (42)$$

In this incorrectly specified model, the true error term ϵ_t may be stated as shown in equation 43.

$$\epsilon_t = \beta_3 AC_t + e_t \qquad (43)$$

[56] Recall that the regression analysis corrections for serial correlation are very similar to the autoregressive process of the transfer function model. For example, a AR(1,0,0) time-series model is equivalent to a regression model that is corrected for first-order serial correlation.

Assuming that AC_t follows a time-related trend, equation 43 may be rewritten with a time-series model of the form:

$$(1-B)^d(1-B^s)^D e_t = \frac{\theta(B)\Theta(B)}{\phi(B)\Phi(B)} \epsilon_t \qquad (44)$$

Referring back to equation 32, one can see that equation 44 is simply the ARIMA component of the transfer function model with ϵ equal to the random shock variable a. This exercise illustrates the primary strength of the transfer function model, namely that the influence of omitted variables is at least partially accounted for by the time-series analysis. Since the model may be more fully specified, the coefficient estimates of the transfer function model will be more efficient than the coefficient estimates of the ordinary regression model. Likewise, the standard errors of the transfer function model will be less biased than the standard errors of the regression model.

CHAPTER 3

EMPIRICAL TESTING

Survey and Data Collection Methodology

Economic theory asserts that the demand for a good or service is a function of its own price, the price of substitute commodities, the behavioral characteristics of the consumer and some other influential socio-economic factors. Applied to public transportation, the demand for mass transit may be postulated as being a function of fare, transit services, costs of driving the automobile, employment and population in the service area, seasonal variations, and non-periodic events such as work stoppages and marketing promotions. By mathematically modeling historical correlations, one may estimate the functional relationships between transit ridership and these explanatory variables. As explained in the preceding chapters, both ARIMA and OLS methods are used in this study to estimate these transit demand functions. However, the primary purpose of this study is to measure the systems' fare elasticities and the demand functions are only a means by which fare elasticities are computed.

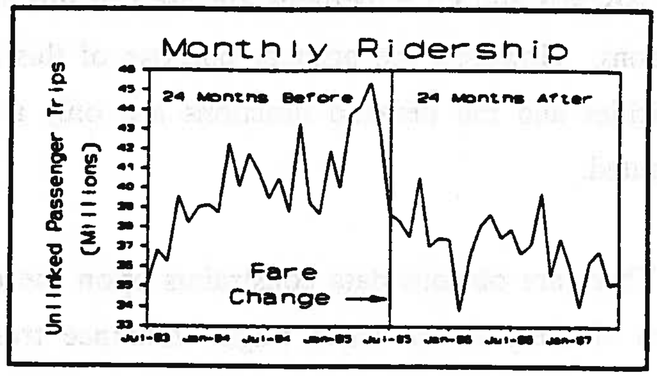
There are obvious data constraints upon the capacity of any model to simulate the effects of every factor which might influence transit demand. Therefore, it becomes necessary to limit the analysis to certain key variables. Also, when data are not available to directly measure primary explanatory variables, like service quality, the model should allow for the use of proxies without adverse impacts on the fare elasticity estimates. Below is a listing of the information needed for the transit demand models proposed in this study.

- Fare Variables
    - Date of Last Fare Change
    - Base Adult Cash Fare
    - Passenger Fare Revenues For Transit Service

- Service Variables
  - Vehicle Revenue Miles
  - Vehicle Revenue Hours
  - Working Days Per Month

- Other Influential Variables
  - Local Employment
  - Local Price Deflator
  - Local Gasoline Price
  - Nature and timing of any extraordinary events which had a significant impact on ridership (intervention variables)

Figure 8. Observation Period



It is essential that the ridership model be able to capture the lag effects of fare changes, since the consumer's time-delayed reaction is a major fare and service policy consideration. Therefore, the analysis must focus on the smallest measurement unit of ridership data readily available, which is the month. The time period of observation was from 24 months before the last fare change to 24 months after the last fare change, for a total of 48 observations, as shown in Figure 8. This would give a view of the market at equilibrium for two years before the fare change and would allow enough time for the market to settle during the 24 months after the fare change. It would also supply the minimum number of observations required by the Box-Jenkins techniques. In practice, certain modifications in the observation period were necessary as very recent fare changes did not have enough observations to conduct the statistical analysis and very old fare changes would not reflect current market conditions.

The data used in this study were obtained from three sources: 1) publications and internal documents of the American Public Transit Association, 2) government and trade periodicals, and 3) information from a special survey which was sent to 189 transit systems in the United States and Canada in May 1988. The date of the system's last fare change was the cardinal determinant of whether the transit system was surveyed. Systems which experienced their last base adult cash fare change before 1982 or after October 1987 were excluded. In the first case, elasticities estimated from fare changes which occurred during the oil embargoes and gasoline price increases of the late 1970s and early 1980s would probably be irrelevant to passenger behavior in the late 1980s, an oil glut period. In the second case, the remaining time from the fare change until May 1988, which was the survey mailing date, was too brief to affirm that the market had reached its long-run equilibrium state. Once the candidate systems were established, the Base Adult Cash Fare was collected from *Transit Fare Summary*[56] for the relevant time period.

The selected transit systems were sent the *Fare Impact Survey*,[57] which requested monthly data for Unlinked Passenger Trips, Vehicle Revenue Miles, Vehicle Revenue Hours and Passenger Fare Revenues For Transit Service during both peak hours and all operating hours for 24 months before and after the system's last fare change. Off-peak data were calculated by subtracting the peak values from the all-hours values.

Since the 48 month period of observation of each transit system varied by the date of their last fare change, each survey had to be customized, via computer program, with the system's name, the correct reporting dates and base adult cash fare. As each transit system returned its survey, the data was compiled and computerized. The response to the survey is given in Table 6. The response rate is good, considering that the survey was extensive and that the completion and return of the survey was entirely voluntary.

[56] American Public Transit Association, various editions.

[57] See Appendix B for a sample copy of the *Fare Impact Survey*.

Table 6. Response of Transit Systems to the Fare Impact Survey

Total number of surveys sent	189
Total number of surveys returned	79
Total response rate	42%
Total number of unuseable surveys returned	27
Total number of useable surveys returned	52
Total useable response rate	28%

Where possible, the reported monthly Unlinked Passenger Trips, Revenue Vehicle Miles and Revenue Vehicle Hours were checked for accuracy by comparing the fiscal year totals to data published in *Transit Operating and Financial Statistics*.[58] If the inconsistency could not be resolved, the offending variable was not used in the analysis. Passenger Fare Revenues For Transit Service was checked against the average fare, which is calculated by dividing Passenger Fare Revenues by Unlinked Passenger Trips. The average fare is generally slightly less than the base adult cash fare and it should exhibit monthly variation. In one case the average fare did not vary, but instead took a constant value. As a fare analysis can not be conducted without a fare variable, this system was omitted from the study. Peak Period Passenger Fare Revenues For Transit Service was checked to ensure that the data was not a constant ratio of the total data. If this was the case, a peak period analysis was not performed since the peak period fare elasticity would be identical to the all-hours fare elasticity.

The last three items, Local Employment, Local Price Deflator and Local Gasoline Price were assembled from the periodical literature. Monthly local employment was taken from *Employment and Earnings*.[59] The Consumer Price Index (CPI-U) of the transit system's

[58] American Public Transit Association, various annual editions.

[59] U.S. Department of Labor, Bureau of Statistics, *Employment and Earnings*, various monthly issues, Table B-8: Employees on nonagricultural payrolls in the States and selected areas by major industry.

headquarters city was used as the Local Price Deflator to deflate the monetary variables.[60] That is, the CPI was used to express fare and gasoline price in constant dollars so that the effects of inflation on the changes in these variables was discounted. The last variable, Local Gasoline Price, was gathered from the weekly periodical *Oil and Gas Journal*.[61] The mid-month data was used to approximate the average monthly gasoline price.

The problems with the CPI, employment and gasoline price variables centered around the fact that not all localities were represented in the data bases of the information sources. As a solution, when local data was unavailable, regional data was used. For instance, the CPI of the West Region – city population of 75,000 to 385,000 was used as the Local Price Deflator for Spokane, Washington. When regional data was not available, the data for a similar city was used. For example, the Buffalo, New York gasoline price was used as the Local Gasoline Price for Albany, New York. Finally, when neither regional nor comparable data was available, the national average was used. The net effect of these substitutions was minimal since the tranfer function methodology used differences, instead of the actual values.

The data was entered on an IBM PC type microcomputer using the LOTUS 1-2-3 spreadsheet software. The elasticity estimations and analyses were then performed with RATS (Regression Analysis of Time Series), a time series analysis and forecasting program.[62]

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[60] U.S. Department of Labor, Bureau of Labor Statistics, *CPI Detailed Report*, various monthly issues, Table 11: Consumer Price Index for All Urban Consumers: Selected areas, all items index.

[61] *Oil and Gas Journal*, various weekly issues with middle-month prices, Table: Gasoline Prices, Approximate prices for self-serve unleaded gasoline, Pump price.

[62] RATS is available from VAR Econometrics, Inc., 1800 Sherman Ave. Suite 612, Evanston, Illinois 60201, telephone (312) 864-8772.

Model Application Results

The ARIMA model specified in Chapter 2 was applied to data from 52 transit systems to estimate the demand functions for fixed-route bus transportation in various cities throughout the United States. The results are presented in Table 7, Bus Ridership Model Results.

In general the economic behavior of the transit riders is well predicted by the model. The corrected coefficient of determination ($\bar{R}^2$) ranges from 0.51 to 0.97, denoting that more than 50 percent and up to 97 percent of the fluctuation in transit ridership is explained by the model. Twenty-five cases have a $\bar{R}^2$ of 0.80 or higher, and for seven cases, the model is able to explain more than 90 percent of the ridership variations.

All coefficients have *a priori* expected signs and are statistically significant at the 90 to 99 percent confidence level in most cases. Coefficients of the fare variable are within the expected range and the lag structures are commonly accepted.

The definitions of the headings and contents of Table 7 are shown below.

LOCATION — The major city in the transit system's service area. Note that the transit systems are listed alphabetically by location.

CONSTANT — The point where the ridership demand function intercepts the Y axis.

FARE — The regression coefficient of the passenger cost variable (FC). In this study, the real average fare was used to approximate FC. Also note that for all variables monthly lags are shown in square brackets to the right of the coefficient and the *t*-statistics are shown in parentheses directly under the coefficient.

SERVICE — The coefficient of the variable used to measure level of bus service at the transit system (SL). Depending on the form of the equation, either revenue vehicle miles (VM), revenue vehicle hours (VH) or working days (WD) per month was used as the service level proxy.

EMPLOYMENT — The coefficient of the market size variable (MC). The monthly employment figure for the major city in the transit's systems service area as defined by the U.S. Department of Labor was used.

GAS PRICE — The coefficient of the alternate mode costs variable (AC). The real (deflated) pump price of major brand, self-service, unleaded gasoline in the area of the transit system's location was used as the alternate mode costs proxy.

INTERVENTION — The coefficient of the binary (0 or 1) variable (I), which is used to account for the effects of certain exceptional, non-periodic incidents which have impacted bus ridership at the system. These events include strikes, free fare promotions, extreme weather and the like.

ERROR STRUCTURE — For transfer function equations, these are the parameters of the ARIMA model used to explain the residual series of the structural regression. The coefficients of the moving average process is shown as MA_{t-i} where *t* is the current month and *i* is the number of months that the moving average polynomial was lagged. Similarly, the coefficient of the autoregressive process is specified as AR_{t-i} where *i* is the number of months that the autoregressive polynomial as lagged. When OLS was used, RHO signifies the coefficient of the first-order autocorrelation correction.

$\bar{R}^2$ — This is the adjusted or corrected coefficient of determination. $\bar{R}^2$ is a measure of the "goodness of fit" of the equation, adjusted for the number of degrees of freedom. The higher $\bar{R}^2$ is, the higher is the degree of overall fit of the estimated regression equation to the data. However, $\bar{R}^2$ is only one of a number of factors that should be considered when evaluating the equation.

D-W — This is value of the Durbin-Watson test which involves the calculation of a test statistic based on the residuals from the regression. The DW statistic will lie in the 0 to 4 range, with a value near 2 indicating no first-order serial correlation.

TIME PERIOD — This is the calendar range over which the actual regression was performed. Whenever possible, the experiment was conducted from 24 months preceding the fare change to 24 months after the fare change.

Table 7. Bus Ridership Model Results

Location	Constant	Fare	Service*	Employment	Gas Price	Intervention	Error Structure	$\bar{R}^2$	D-W	Time Period
Albany, NY	0.00013	-0.46 [t-1] (3.42)	WD 0.43 (3.30)	0.80 (1.54)			MA_{t-1} -0.95 (7.98) MA_{t-12} -0.49 (2.00)	0.57	1.94	1984:6 to 1987:3
Alexandria, VA	-0.00902	-0.14 (2.29) -0.28 [t-2] (4.31)	VM 1.00 (15.45)	0.73 [t-5] (2.10)				0.91	2.03	1985:12 to 1988:4
Allentown, PA	-0.00322	-0.75 (2.60)	VM 0.96 (2.89)		0.62 [t-2] (3.42)			0.70	2.23	1986:7 to 1988:4
Appleton, WI	0.00280	-0.26 (2.86)	VM 0.60 (3.81)	2.77 [t-3] (1.73)			AR_{t-1} -0.58 (2.73)	0.61	1.99	1986:6 to 1988:5
Atlanta, GA	-4.28	-0.28 (2.72)	VM 1.31 (5.33)		0.12 [t-4] (2.02)			0.51	1.98	1985:10 to 1988:5
Baltimore, MD	0.002	-0.49 (3.40)	VM 0.99 (7.16)				AR_{t-1} -0.42 (2.18) MA_{t-12} -0.60 (2.40)	0.78	2.15	1986:1 to 1988:4
Binghamton, NY	-8.32	-0.70 (10.95)	VM 0.86 (8.89)	1.80 [t-4] (2.45)	0.23 [t-3] (1.46)		RHO 0.90 (10.74)	0.93	1.63	1985:7 to 1988:4

NOTES: Functional forms are LOG-LOG.
Monthly lags are shown in square brackets to right of coefficient; t-statistics are shown in parentheses under coefficient.
* - Service variables are either Vehicle Hours (VH), Vehicle Miles (VM), or Working Days per month (WD).
MA_{t-i} is the moving average polynomial lagged i months and AR_{t-i} is the autoregressive polynomial lagged i months of the transfer function.
RHO is the correction for first-order serial correlation.

Table 7. Bus Ridership Model Results

Location	Constant	Fare	Service*	Employment	Gas Price	Intervention	Error Structure	$\bar{R}^2$	D-W	Time Period
Boone, NC	-19.47	-0.53 (5.67)	VM 0.36 (1.93)	5.39 [t-3] (2.18)		0.75 (4.35) ski bus 12/85 to 2/86 = 1		0.81	1.63	1985:10 to 1988:5
Buffalo, NY	-0.00238	-0.50 (3.27)	VM 1.04 (5.11)				MA_{t-1} -0.91 (8.58) MA_{t-12} -0.82 (3.67)	0.84	2.09	1984:12 to 1987:10
Chattanooga, TN	-0.69	-0.34 (4.75)	VM 0.91 (7.02)		0.44 [t-4] (3.67)	0.06 (2.48) June = 1 peak month	RHO 0.54 (3.80)	0.88	1.90	1983:12 to 1987:6
Cincinnati, OH		-0.47 (2.87) -0.27 [t-3] (1.98)	VH 1.00 (9.00)	1.64 (2.27)			MA_{t-1} -0.65 (2.65) MA_{t-12} -0.69 (3.08)	0.80	1.86	1984:7 to 1987:2
Clearwater, FL		-0.25 [t-1] (3.91) -0.23 [t-3] (3.19)	VM 1.20 (11.07)	1.28 [t-1] (2.13)	0.12 (1.87)		AR_{t-1} 0.43 (2.12)	0.74	1.83	1985:3 to 1987:9

NOTES: Functional forms are LOG-LOG.
Monthly lags are shown in square brackets to right of coefficient; t-statistics are shown in parentheses under coefficient.
* - Service variables are either Vehicle Hours (VH), Vehicle Miles (VM), or Working Days per month (WD).
MA_{t-i} is the moving average polynomial lagged i months and AR_{t-i} is the autoregressive polynomial lagged i months of the transfer function.
RHO is the correction for first-order serial correlation.

Table 7. Bus Ridership Model Results

Location	Constant	Fare	Service*	Employment	Gas Price	Intervention	Error Structure	$\bar{R}^2$	D-W	Time Period
Dallas, TX	3.91	-0.13 (1.77)	VH 0.93 (6.92)					0.91	1.76	1985:10 to 1987:9
Denver, CO	-0.00218	-0.56 (20.60)	VM 0.19 (6.96)		0.18 [t-3] (10.00)		AR_{t-1} -0.87 (28.98) AR_{t-2} -0.79 (41.34) MA_{t-12} -12.01 (1.97)	0.92	2.31	1986:12 to 1988:4
Des Plaines, IL		-0.12 [t-3] (1.75)	VM 0.24 (2.19)	1.08 (1.91) 1.17 [t-1] (2.28)			MA_{t-1} -1.15 (15.62)	0.73	1.81	1985:6 to 1988:1
Detroit, MI	4.90	-0.25 (3.18)	VM 0.63 (13.21)			-0.22 (4.44) Strike 12/83		0.92	1.48	1981:7 to 1985:6
El Paso, TX	0.00319	-0.29 (2.54)	VM 0.77 (3.15)	2.48 [t-3] (1.67)	0.21 [t-2] (1.58)		AR_{t-1} -0.48 (2.24)	0.50	1.99	1986:2 to 1988:4
Eugene, OR	-0.0133	-0.18 (1.89)	VH 0.58 (6.47) VH 0.20 [t-1] (2.05)	2.98 (5.40)	0.27 [t-3] (4.53)	0.05 (2.94) free fare, 2nd wk Aug	AR_{t-1} -0.64 (2.67)	0.84	2.16	1986:11 to 1988:3

NOTES: Functional forms are LOG-LOG.

Monthly lags are shown in square brackets to right of coefficient; t-statistics are shown in parentheses under coefficient.

* - Service variables are either Vehicle Hours (VH), Vehicle Miles (VM), or Working Days per month (WD).

MA_{t-i} is the moving average polynomial lagged i months and AR_{t-i} is the autoregressive polynomial lagged i months of the transfer function.

RHO is the correction for first-order serial correlation.

Table 7. Bus Ridership Model Results

Location	Constant	Fare	Service*	Employment	Gas Price	Intervention	Error Structure	$\bar{R}^2$	D-W	Time Period
Everett, WA	-0.00281	-0.43 (1.82)	VM 1.17 (2.45)				AR_{t-1} -0.36 (2.35) MA_{t-12} -0.74 (2.74)	0.51	2.25	1982:9 to 1985:6
Flint, MI	-0.339	-0.59 (2.98)	VM 0.62 (5.71)	0.60 (1.36)	0.31 [t-4] (2.13)		RHO 0.64 (4.92)	0.87	1.59	1984:7 to 1988:1
Fresno, CA	0.00127	-0.31 (4.99)	VM 0.63 (3.86)	1.47 [t-4] (3.44)			AR_{t-1} -1.06 (6.83) AR_{t-2} -0.77 (4.94)	0.74	2.00	1985:2 to 1987:6
Fort Wayne, IN (fare decrease)	5.32	-0.12 [t-5] (1.77)	VM 0.55 (8.30)					0.90	1.81	1985:6 to 1987:12
Grand Rapids, MI	-0.00043	-0.43 (6.89)	VM 0.77 (5.63)		0.10 [t-3] (1.50)		AR_{t-1} -0.49 (2.74)	0.84	1.99	1985:1 to 1987:7
Grand Rapids, MI (peak period)	0.00092	-0.19 (3.02)	VM 0.84 (6.40)		0.10 [t-3] (1.68)		AR_{t-1} -0.38 (2.02)	0.80	1.87	1985:1 to 1987:7
Grand Rapids, MI (off-peak period)	-0.00164	-0.53 (7.80)	VM 0.80 (4.84)				AR_{t-1} -0.51 (3.09)	0.79	2.19	1985:1 to 1987:7

NOTES: Functional forms are LOG-LOG.
Monthly lags are shown in square brackets to right of coefficient; t-statistics are shown in parentheses under coefficient.
* - Service variables are either Vehicle Hours (VH), Vehicle Miles (VM), or Working Days per month (WD).
MA_{t-i} is the moving average polynomial lagged i months and AR_{t-i} is the autoregressive polynomial lagged i months of the transfer function.
RHO is the correction for first-order serial correlation.

Table 7. Bus Ridership Model Results

Location	Constant	Fare	Service*	Employment	Gas Price	Intervention	Error Structure	$\bar{R}^2$	D-W	Time Period
Gretna, LA	-0.00019	-0.35 (3.10)	VM 0.85 (5.03)	1.81 [t-1] (1.64)			AR_{t-1} -0.35 (1.52)	0.77	1.84	1986:5 to 1988:5
Honolulu, HI	-0.00050	-0.65 (5.99)	VM 0.72 (8.76)				MA_{t-12} -0.51 (2.05) MA_{t-24} 0.77 (1.73)	0.80	1.93	1983:7 to 1986:5
Kansas City, MO	7.52	-0.51 (4.32)	VM 0.31 (2.16)		0.43 [t-2] (2.89)		RHO 0.40 (2.67)	0.92	1.93	1981:4 to 1984:12
Lancaster, PA		-0.43 (2.94)	VM 0.68 (5.01)	1.12 [t-1] (1.47)			MA_{t-1} -0.64 (3.82)	0.79	1.75	1984:9 to 1987:6
Lincoln, NE	-0.00779	-0.50 (3.26)	VM 0.62 [t-3] (3.12)		0.25 [t-3] (1.86)		AR_{t-1} -0.75 (4.24)	0.55	2.24	1984:2 to 1986:8
Los Angeles, CA	3.66	-0.23 (5.83)	VM 0.84 (11.08)				RHO 0.82 (11.32)	0.87	2.06	1983:8 to 1987:6
Los Angeles, CA (peak period)	-0.00574	-0.22 (6.11)	VM 1.00 (21.68)				MA_{t-12} -0.55 (2.86)	0.92	2.00	1984:8 to 1987:6
Los Angeles, CA (off-peak period)	4.98	-0.24 (4.66)	VM 0.74 (9.24)				RHO 0.87 (12.70)	0.80	2.08	1983:8 to 1987:6

NOTES: Functional forms are LOG-LOG.
Monthly lags are shown in square brackets to right of coefficient; t-statistics are shown in parentheses under coefficient.
* - Service variables are either Vehicle Hours (VH), Vehicle Miles (VM), or Working Days per month (WD).
MA_{t-l} is the moving average polynomial lagged l months and AR_{t-l} is the autoregressive polynomial lagged l months of the transfer function.
RHO is the correction for first-order serial correlation.

Table 7. Bus Ridership Model Results

Location	Constant	Fare	Service*	Employment	Gas Price	Intervention	Error Structure	$\bar{R}^2$	D-W	Time Period
Madison, WI	-0.00377	-0.18 [t-1] (2.46) -0.22 [t-3] (2.34)	VM 0.94 (6.76)	2.68 [t-2] (2.68)	0.16 [t-6] (2.79)		AR_{t-1} -0.60 (2.48)	0.83	1.89	1986:9 to 1988:4
Nashville, TN	-0.00524	-0.24 [t-1] (3.25) -0.29 [t-2] (3.86)	VM 0.69 (4.16)		0.30 [t-5] (3.25)		AR_{t-1} -1.14 (5.39) AR_{t-2} -0.35 (1.65)	0.82	1.97	1984:6 to 1986:9
Oceanside, CA	0.55	-0.35 (2.64)	VM 0.86 (6.74)		0.22 (2.54)	-0.0035 (2.20) month as Jan = 1,..., Dec = 12	RHO 0.59 (3.74)	0.68	1.76	1984:8 to 1987:6
Oshkosh, WI		-0.17 [t-1] (3.09)	VM 1.97 (11.21)	2.81 [t-5] (1.63)	0.36 [t-4] (2.74)		AR_{t-1} 0.23 (1.11)	0.86	1.86	1985:8 to 1987:12
Phoenix, AZ	-0.00399	-0.32 (1.86)	VM 1.17 (4.25)		0.65 [t-2] (2.34)		AR_{t-1} -0.83 (4.25) AR_{t-2} -0.33 (1.67)	0.66	1.86	1983:12 to 1986:6
Portland, OR	0.00064	-0.39 (4.30)	VM 0.48 (2.35)	0.95 [t-2] (1.48)	0.33 [t-4] (2.51)	-0.39 (2.34) light rail start 9/86	AR_{t-1} -0.50 (2.64)	0.64	1.99	1985:3 to 1987:8

NOTES: Functional forms are LOG-LOG.
Monthly lags are shown in square brackets to right of coefficient; t-statistics are shown in parentheses under coefficient.
* - Service variables are either Vehicle Hours (VH), Vehicle Miles (VM), or Working Days per month (WD).
MA_{t-i} is the moving average polynomial lagged i months and AR_{t-i} is the autoregressive polynomial lagged i months of the transfer function.
RHO is the correction for first-order serial correlation.

Table 7. Bus Ridership Model Results

Location	Constant	Fare	Service*	Employment	Gas Price	Intervention	Error Structure	$\bar{R}^2$	D-W	Time Period
Portland, OR (peak period)		-0.19 (1.75)	VM 0.48 (1.78)	1.28 [t-2] (1.43)	0.36 [t-5] (1.57)	-0.52 (2.47) light rail start 9/86		0.30	2.38	1985:3 to 1987:8
Portland, OR (off-peak period)	0.00144	-0.43 (3.69)	VM 0.72 (5.38)	0.68 [t-2] (2.21)	0.53 [t-4] (3.27)	-0.40 (2.10) light rail starts 9/86	MA_{t-12} -1.01 (4.20) MA_{t-24} -1.47 (2.38)	0.57	2.28	1985:2 to 1987:8
Richmond, VA	-0.00266	-0.62 (2.43)	VM 1.29 (7.47)	1.67 [t-1] (2.03)	0.16 [t-2] (1.96)			0.70	2.58	1985:2 to 1987:10
Riverside, CA	0.00499	-0.12 (3.96)	VM 0.82 (7.81)	0.91 (1.86)			AR_{t-1} -0.85 (6.58)	0.76	2.05	1986:3 to 1988:4
Sacramento, CA	0.00006	-0.16 (7.58)	VM 1.05 (17.26)		0.22 (3.98)	-0.017 (5.26) light rail start 3/87	AR_{t-1} -0.78 (5.75) MA_{t-12} -1.70 (6.70)	0.84	2.04	1985:4 to 1988:1
Sacramento, CA (peak period)	0.00097	-0.22 (5.84)	VM 0.71 (12.68)		0.14 (1.43)	-0.015 (4.77) light rail start 3/87	AR_{t-1} -0.64 (4.25) MA_{t-12} -1.22 (6.79)	0.83	1.88	1985:4 to 1988:1

NOTES: Functional forms are LOG-LOG.
Monthly lags are shown in square brackets to right of coefficient; t-statistics are shown in parentheses under coefficient.
* - Service variables are either Vehicle Hours (VH), Vehicle Miles (VM), or Working Days per month (WD).
MA_{t-l} is the moving average polynomial lagged l months and AR_{t-l} is the autoregressive polynomial lagged l months of the transfer function.
RHO is the correction for first-order serial correlation.

Table 7. Bus Ridership Model Results

Location	Constant	Fare	Service*	Employment	Gas Price	Intervention	Error Structure	$\bar{R}^2$	D-W	Time Period
Sacramento, CA (off-peak period)	-0.00183	-0.13 (2.75)	VM 1.37 (8.46)		0.35 (2.95)	-0.014 (3.01) light rail start 3/87	AR_{t-1} -0.55 (3.20) MA_{t-12} -1.32 (6.49)	0.66	1.86	1985:4 to 1988:1
San Diego, CA		-0.27 (1.85)	VM 0.49 (3.67)	4.85 (2.61)			MA_{t-1} -0.73 (4.67) MA_{t-12} -1.24 (4.05)	0.66	1.72	1986:2 to 1988:4
San Francisco, CA	-0.00276	-0.15 (2.28)	VM 1.13 (9.60)	2.15 [t-1] (3.10)				0.88	1.86	1983:5 to 1985:6
San Francisco, CA (peak-period)	0.00132	-0.14 (3.09)	VM 0.88 (15.69)				MA_{t-1} -0.50 (2.08)	0.96	1.60	1983:8 to 1985:6
San Francisco, CA (off-peak period)		-0.31 (2.60)	VM 1.64 (3.20)	4.25 [t-1] (2.78)				0.80	2.00	1983:9 to 1985:6
San Jose, CA	-0.00635	-0.46 (2.17)	VM 0.82 (2.92)	1.97 [t-1] (1.53)	0.34 [t-3] (2.15)			0.52	2.20	1986:5 to 1988:4
Sarasota, FL	-0.00114	-0.21 (2.67)	VM 0.92 (5.46)	1.57 [t-2] (2.13)			MA_{t-12} -0.46 (1.96)	0.68	2.13	1986:1 to 1988:5

NOTES: Functional forms are LOG-LOG.
Monthly lags are shown in square brackets to right of coefficient; t-statistics are shown in parentheses under coefficient.
* - Service variables are either Vehicle Hours (VH), Vehicle Miles (VM), or Working Days per month (WD).
MA_{t-l} is the moving average polynomial lagged l months and AR_{t-l} is the autoregressive polynomial lagged l months of the transfer function.
RHO is the correction for first-order serial correlation.

Table 7. Bus Ridership Model Results

Location	Constant	Fare	Service*	Employment	Gas Price	Intervention	Error Structure	$\bar{R}^2$	D-W	Time Period
Seattle, WA	-0.00133	-0.27 (2.35)	VH 1.01 (10.26)				AR_{t-1} -0.48 (2.76) MA_{t-12} -0.62 (2.96)	0.86	1.81	1984:4 to 1987:1
South Bend, IN		-0.26 (4.58)	VM 0.79 (4.68)	0.53 (1.48)	0.52 [t-1] (3.88)		MA_{t-1} -0.76 (4.29)	0.66	1.95	1982:3 to 1984:12
South Daytona, FL	0.00319	-0.42 (2.88)	VM 0.70 (4.11)	1.77 [t-2] (1.80)	0.15 (1.85)		AR_{t-1} -0.34 (1.58)	0.61	1.96	1986:2 to 1988:4
Spokane, WA	0.00099	-0.30 (5.69) 0.44 [t-1] (3.15) denom. lag	VM 0.13 (4.10) 0.11 [t-1] (3.49)		0.22 [t-1] (2.27)		AR_{t-1} -1.05 (7.87) AR_{t-2} -0.75 (5.36) MA_{t-12} -1.14 (5.81)	0.69	2.11	1983:5 to 1985:12
Spokane, WA (peak period)		-0.32 [t-3] (4.18)	VM 0.19 (2.50)		0.38 [t-1] (2.97)		MA_{t-1} -0.78 (4.60)	0.59	1.95	1983:5 to 1985:12
Spokane, WA (off-peak period)	-0.00293	-0.28 (3.12) -0.46 [t-1] (5.36)	VM 0.19 (4.07) 0.13 [t-1] (2.77)		0.42 [t-1] (2.22)		MA_{t-12} -0.96 (5.11)	0.64	2.00	1983:3 to 1985:12

NOTES: Functional forms are LOG-LOG.
Monthly lags are shown in square brackets to right of coefficient; t-statistics are shown in parentheses under coefficient.
* - Service variables are either Vehicle Hours (VH), Vehicle Miles (VM), or Working Days per month (WD).
MA_{t-i} is the moving average polynomial lagged i months and AR_{t-i} is the autoregressive polynomial lagged i months of the transfer function.
RHO is the correction for first-order serial correlation.

Table 7. Bus Ridership Model Results

Location	Constant	Fare	Service*	Employment	Gas Price	Intervention	Error Structure	$\bar{R}^2$	D-W	Time Period
Springfield, MO	-0.00135	-0.48 (6.57)	VM 0.23 (1.55)		0.16 [t-1] (1.73)		AR_{t-1} -0.35 (2.02) AR_{t-2} -0.55 (3.15) MA_{t-12} 0.56 (2.31)	0.65	2.06	1982:5 to 1984:12
State College, PA	-17.53	-0.64 (4.57)	VM 1.62 (4.37)	1.96 (2.29)	0.78 [t-4] (4.01)			0.89	1.73	1984:11 to 1987:2
Tacoma, WA	0.00291	-0.43 (4.70)	VM 0.85 (2.73)		0.45 [t-1] (2.63)		AR_{t-1} -0.50 (2.50) AR_{t-2} -0.57 (2.88)	0.63	1.79	1983:5 to 1985:12
Toledo, OH	0.184	-0.85 (29.54)	VM 0.87 (13.96)				RHO 0.26 (1.56)	0.97	2.11	1985:3 to 1988:4
Williamsport, PA	-0.00015	-0.30 (2.52)	VM 0.83 (10.18)	0.84 [t-3] (2.56)			MA_{t-12} -0.68 (2.66)	0.75	2.32	1985:10 to 1988:5
W. Palm Beach, FL	-5.99	-0.60 (2.92)	VM 0.92 (7.19)	1.07 (2.53)			RHO 0.91 (12.33)	0.86	2.38	1985:4 to 1988:1

NOTES: Functional forms are LOG-LOG.
Monthly lags are shown in square brackets to right of coefficient; t-statistics are shown in parentheses under coefficient.
* - Service variables are either Vehicle Hours (VH), Vehicle Miles (VM), or Working Days per month (WD).
MA_{t-i} is the moving average polynomial lagged i months and AR_{t-i} is the autoregressive polynomial lagged i months of the transfer function.
RHO is the correction for first-order serial correlation.

Calculation of Fare Elasticities

The goal of this study was to determine the industry-wide response of bus patrons to fare increases. The demand equations are merely the tool which permits this measurement. Since all of the equations are in the double log form, that is both the dependent and independent variables have a natural logarithmic transformation, for most equations the fare coefficient itself is the elasticity value. However, with the transfer function model there are several cases which need special consideration. First is the presence of lagged numerator parameters in the fare variable. In this case, the long-run fare elasticity is calculated by the summation of the current and lagged fare coefficients. The second is the presence of lagged denominator parameters. Here, the long-run fare elasticity is calculated by dividing the current numerator coefficient by one minus the denominator coefficient.

The generalized fare elasticities presented in this study are subject to certain unavoidable limitations which necessitates their judicious use. Some special qualifications are outlined below.

- The elasticities are based on system-wide bus ridership. Therefore, their application to any particular bus route or type of service is debateable.

- The fare structure was assumed to have remained constant. Comprehensive changes in the gross fare structure, for example the introduction of tokens, calls for a re-examination of the elasticity.

- The focus of this study is to estimate fare elasticities. While the estimated transit demand functions provide a set of coefficients which may allow the derivation of elasticities of the service, employment, gas price and intervention variables, the formulation of these other elasticities was not the aim of this research. They are merely a by-product of it. The coefficients are statistically significant and of the proper signs and magnitudes, but their proper use requires further investigation.

Fare Elasticity Findings

This study attempts to answer the following questions:

- What are the effects of fare changes on bus ridership?

- Have there been any changes in the fare elasticity levels since the completion of the Simpson-Curtin study some 40 years ago?

- Is the demand for bus transit less responsive to fare changes during peak travel periods?

- Do fare elasticities differ between large and small cities?

- Do the initial base fare levels have on influence on a transit system's fare elasticity level?

The findings on fare elasticities of bus service for fifty-two transit systems under study are presented in Table 8 (all day average) and Table 9 (peak off-peak differential). Briefly, the results are as follows:

- The fare elasticity for all systems averages at -0.40, notably higher than the Simpson-Curtin formula of -0.33 which has been widely used by transit managers.

- The elasticity levels of individual transit systems, however, vary widely, from -0.12 for Riverside, Calif. to -0.85 for Toledo, Ohio. The local population work places, income, driving conditions, transit services, parking costs, etc., cause different levels of sensitivity of travellers to fare changes. In any event, the large variation clearly illustrates the danger of applying the Simpson-Curtin rule to all areas.

Table 8. Transit Fare Elasticity Estimates of 52 Transit Systems

	CITY	URBAN AREA POPULATION	FARE ELASTICITY	t-STAT	R SQUARED	FARE ELAST GROUP MEANS
	BUS SERVICES IN URBANIZED AREAS WITH MORE THAN 1 MILLION POPULATION					
1	Los Angeles, CA	9,479,436	-0.231	5.83	0.87	
2	Des Plaines, IL	6,779,799	-0.117	1.75	0.73	
3	Detroit, MI	3,809,327	-0.247	3.18	0.92	
4	San Francisco, CA	3,190,698	-0.151	2.28	0.88	
5	Alexandria, VA	2,763,105	-0.412	2.29	0.91	
6	Dallas, TX	2,451,390	-0.134	1.77	0.91	
7	Baltimore, MD	1,755,477	-0.495	3.40	0.78	
8	San Diego, CA	1,704,352	-0.270	1.85	0.66	
9	Oceanside, CA	1,704,352	-0.350	2.64	0.68	
10	Atlanta, GA	1,613,357	-0.277	2.72	0.51	
11	Phoenix, AZ	1,409,279	-0.321	1.86	0.66	-0.361
12	Seattle, WA	1,391,535	-0.266	2.35	0.86	(0.154)*
13	Everett, WA	1,391,535	-0.429	1.82	0.51	
14	Denver, CO	1,352,070	-0.562	20.60	0.92	
15	San Jose, CA	1,243,952	-0.460	2.17	0.52	
16	Cincinnati, OH	1,123,412	-0.738	1.98	0.80	
17	Kansas City, MO	1,097,793	-0.511	4.32	0.92	
18	Gretna, LA	1,078,299	-0.354	3.10	0.77	
19	Portland, OR	1,026,144	-0.387	4.30	0.64	
20	Buffalo, NY	1,002,285	-0.503	3.27	0.84	
	BUS SERVICES IN URBANIZED AREAS WITH LESS THAN 1 MILLION POPULATION					
21	Sacramento, CA	796,266	-0.162	7.58	0.84	
22	Riverside, CA	705,175	-0.119	3.96	0.76	
23	Honolulu, HI	582,463	-0.652	5.99	0.80	
24	St. Petersburg, FL	520,912	-0.478	3.19	0.74	
25	Nashville, TN	518,325	-0.527	3.25	0.82	
26	Richmond, VA	491,627	-0.624	2.43	0.70	
27	Albany, NY	490,015	-0.456	3.42	0.57	
28	West Palm Beach, FL	487,044	-0.605	2.92	0.86	
29	Toledo, OH	485,440	-0.855	29.54	0.97	
30	El Paso, TX	454,159	-0.294	2.54	0.50	
31	Tacoma, WA	402,077	-0.432	4.70	0.63	
32	Allentown, PA	381,734	-0.747	2.60	0.70	
33	Grand Rapids, MI	374,744	-0.430	6.89	0.84	
34	Flint, MI	331,931	-0.585	2.98	0.87	
35	Fresno, CA	331,551	-0.311	4.99	0.74	
36	Sarasota, FL	305,431	-0.214	2.67	0.68	
37	Chattanooga, TN	301,515	-0.341	4.75	0.88	-0.430
38	Spokane, WA	266,709	-0.527	3.15	0.69	(0.189)*
39	Fort Wayne, IN	236,479	-0.116	1.77	0.90	
40	South Bend, IN	226,331	-0.261	4.58	0.66	
41	Madison, WI	213,675	-0.401	2.34	0.83	
42	Eugene, OR	182,495	-0.184	1.89	0.84	
43	Lincoln, NE	173,550	-0.500	3.26	0.55	
44	South Daytona, FL	170,749	-0.423	2.88	0.61	
45	Binghamton, NY	161,132	-0.704	10.95	0.93	
46	Lancaster, PA	157,385	-0.428	2.94	0.79	
47	Appleton, WI	142,151	-0.255	2.86	0.61	
48	Springfield, MO	139,030	-0.481	8.57	0.65	
49	Williamsport, PA	58,650	-0.299	2.52	0.75	
50	Oshkosh, WI	52,958	-0.167	3.09	0.86	
51	State College, PA	51,298	-0.642	4.57	0.89	
52	Boone, NC	Non-UZA	-0.528	5.66	0.81	
	ALL SYSTEMS:					-0.403 (0.179)*

* - Standard Deviation

Source: American Public Transit Association

Table 9. Fare Elasticity: Peak and Off-Peak Travel

Urbanized Area	Peak		Off Peak		Population
Spokane, WA	-0.32		-0.73		266,709
Grand Rapids, MI	-0.29		-0.49		374,744
Sacramento, CA[1]	-0.22		-0.14		796,266
GROUP I AVERAGE[2]	-0.27	[0.04]	-0.45	[0.30]	1 million and less
Portland, OR/WA	-0.20		-0.58		1,026,144
San Francisco, CA[3]	-0.14		-0.31		3,190,698
Los Angeles, CA	-0.21		-0.29		9,479,436
GROUP II AVERAGE[2]	-0.18	[0.04]	-0.39	[0.16]	1 million and more
ALL SYSTEMS AVERAGE[2]	-0.23	[0.06]	-0.42	[0.22]	

Notes: 1. Light rail initiated March 1987, which was during the observation period.

2. The standard deviations of the group and total means are contained in square brackets.

3. Transit system serves Marin and Sonoma counties.

- As a group, the average fare elasticity for large cities with more than 1 million population is -0.36, significantly less (in absolute value) than the elasticity of -0.43 estimated for smaller cities. This indicates that transit users in small cities and rural areas are more sensitive to fare increases.[63]

Analysis of the elasticity differentials between peak and off-peak travel are constrained by the limited number of observations: only six transit systems were able to provide peak/off-peak data for the study. Nevertheless, certain patterns clearly emerged, as follows:

[63] Attempts to identify a continuous functional relationship between fare elasticities city sizes and other relevant factors were unsuccessful.

- The average fare elasticity during off-peak hours is -0.42, about twice as high as that during peak hours of value of -0.23.[64] This relationship holds true for both population groups. For cities with more than one million population, the average peak-hour elasticity is -0.18, comparing to the off peak elasticity of -0.39. For smaller cities and rural areas, the numbers are -0.27 and -0.46, for peak and off-peak hours, respectively.

Implications of the Effects of Fare Changes on Bus Ridership and Fare Revenues

For a very small change in fare, fare elasticity measures the ridership shrinkage ratio. In other words, fare elasticity of -0.40 implies a shrinkage ratio of -0.40, or a one percent increase in fare would result in a 0.4 percent decrease (shrinkage) in bus ridership.

However, for larger fare increases, the elasticity and shrinkage levels usually do not coincide. The following formula is used to estimate the new ridership levels as a result of larger fare increases:[65]

$$R_2 = \exp\left[\varepsilon \times \ln\left(\frac{F_2}{F_1}\right) + \ln(R_1)\right] \qquad (45)$$

where ε : fare elasticity

R_2, F_2 : the new ridership and fare levels

R_1, F_1 : the existing ridership and fare levels

exp : the exponential function

ln : the natural logarithmic function

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[64] Patrick Mayworm et al. in *Patronage Impacts of Changes in Transit Fares and Services*, page 85, surveyed existing studies and arrived at the adjustment factors of 0.59 for all-hour to peak conversion and 1.38 for all hour to off-peak conversion. Using these factors, our peak and off-peak elasticities would be -0.24 and -0.55, respectively.

[65] See Chapter 5 for more detailed explanations.

For a fare elasticity of -0.4, the effects of fare increases on ridership and total fare revenues are shown in Table 10.[66]

Table 10.
Effects of Fare Increases on Ridership and Revenues at Fare Elasticity = -0.40

Fare Increase	Ridership Loss	Revenue Increase
10%	3.7%	5.9%
20%	7.0%	11.6%
30%	10.0%	17.0%
40%	12.6%	22.4%
50%	15.0%	27.5%

[66] Assuming that the fare structures and all other influential factors remain unchanged.

CHAPTER 4

USE OF TRANSFER FUNCTION MODEL BY TRANSIT SYSTEMS

This report provides estimates of the general demand functions and fare elasticities of various American transit systems and the national averages. Frequently, transit analysts are required to estimate and update the demand functions for their specific transit systems, and to analyze the expected impacts of fare changes on specific bus or rail routes. The transfer function model can be a valuable tool in accomplishing these objectives. This chapter will briefly provide some guidance and advice concerning the use of this model.

Transfer Function Estimation

The estimation of the transfer function model is only slightly more complicated than the estimation of a regression model. Of course, a computer with an appropriate time-series statistical software package is needed.[67] A step-by-step application of the methodology used to estimate the fare elasticities presented in this paper is given below. The reader should understand that a full and complete explanation of the transfer function model is not possible here and is, therefore, urged to refer to other sources.

STEP 1. **Specify the Theoretical Explanatory Structure of the Model.** This most important step entails specifying the variables to be included as regressors, the signs and approximate magnitudes of the coefficients, and the measurement or proxy for each variable. The selection of variables should be based on the economic concepts of causality, complementarity and substitution. The hypotheses and results offered in this paper and in other studies may provide some guidance regarding which variables are the major determinants of ridership. The researcher should be well-enough acquainted with the

[67] A review of statistical software for IBM-standard microcomputers is Robin Raskin, "Statistical Software for the PC: Testing for Significance," *PC Magazine*, Volume 8 Number 5 (March 14, 1989), 94-310.

economic, statistical and data requirements and limitations of transit analysis in order to determine the interrelationships among the independent variables, which proxies are the best measurements of the theoretically desired variables, and what priors can be imposed. Also, the researcher should be familiar with the qualitative information so as to detect the effects of any anomalies such as work stoppages, the opening of new highways or rail lines or special marketing promotions on the ridership series. Regarding time series analysis, the selection of explanatory variables generally is limited to those variables for which four or more years of *comparable* monthly or quarterly data is available, since the Box-Jenkins method requires at least 40 to 50 observations.[68] Properly executed, this first step will alleviate many of the problems encountered in econometric analysis. However, the researcher must understand that theory development is an on-going process and that the initial hypothesis will be revised throughout the experiment.

STEP 2. **Collect and compile time series data for the dependent and independent variables which will be used in the analysis.** Monthly transit data such as ridership, revenues and vehicle miles and hours usually must be obtained directly from the transit system's internal records. Economic data such as employment, population and income can be obtained from government publications. Some of this data is computerized already and these databases are available for a small fee. Other data, such as gasoline price, may be reported in trade journals and other types of periodical literature. The data should be thoroughly checked for accuracy.

STEP 3. **Plot the ridership series and its autocorrelations and examine these plots for transformation, differencing and seasonality.** Ridership will be non-stationary

[68] C.W.J. Granger and Paul Newbold, *Forecasting Economic Time Series*.

and must be converted into a stationary time series. Ridership will most likely become stationary after performing the log transformation and taking the first and twelfth differences. A graphical representation of this procedure is shown in Figure 9a-Figure 9h. First, the raw data is graphed, as shown in Figure 9a. Notice that the ridership series exhibits both consecutive and seasonal trends. In Figure 9g one can see that both trends have been removed by the natural log transformation and the differencings. Usually, a plot of the transformed data is sufficient to determine if the trend has been removed and a plot of the sample autocorrelations will show whether the lag correlations are invariant with respect to time. Figure 9h shows the sample autocorrelation function of the stationary series. Note that the autocorrelations (ρ_k) of a stationary series will die out quickly as the lag k increases. Although high autocorrelations at very low lags are not evidence of non-stationarity,[69] pay particular attention to peaks or troughs at regular intervals, as shown in Figure 9f, as they indicate seasonality and, therefore, the need for seasonal differencing. Occasionally second consecutive differencing is needed; however, superfluous differencing of a stationary series will only generate another stationary series with an altered, negative autocorrelation pattern. Obviously this is to be avoided as it merely serves to unnecessarily complicate the model.[70]

STEP 4. **Calculate and plot the cross correlations of the ridership series and the proposed explanatory variables.** Examination of these plots, along with the economic theory advanced in Step 1, will help identify the explanatory variables and the orders of their numerator and denominator parameters in the structural component of the transfer function model. As shown in

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[69] Vandaele, p.66.

[70] Nelson, p.76.

Figure 9a. Unlinked Trips

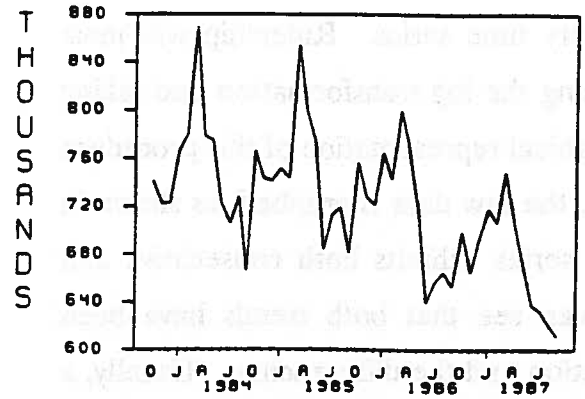


Figure 9b. Sample Autocorrelation Function

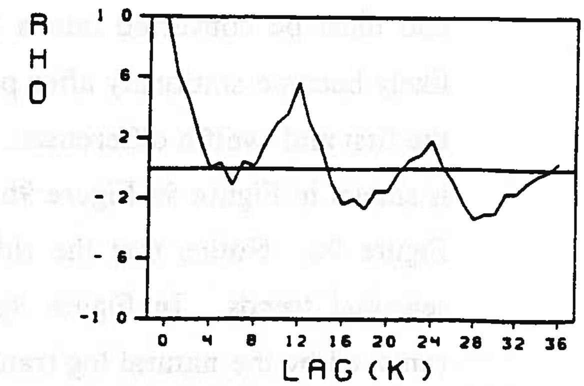


Figure 9c. Log Transformation

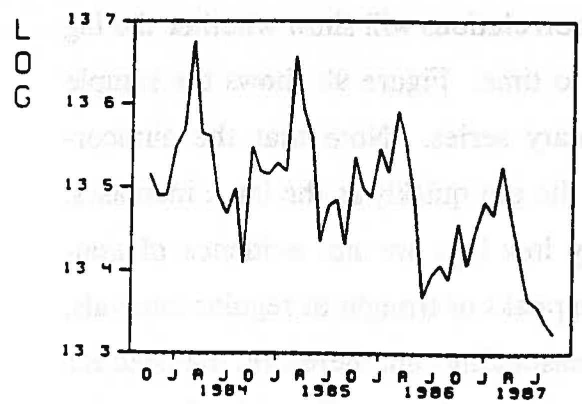


Figure 9d. Sample Autocorrelation Function

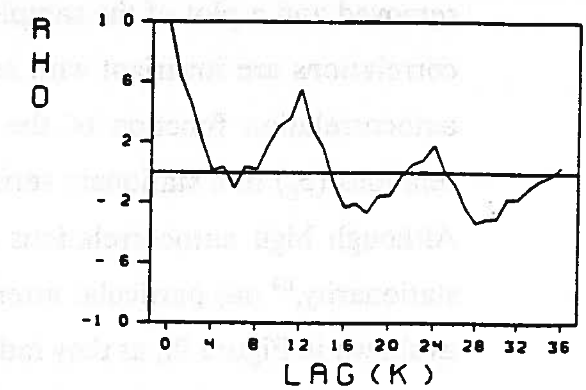


Figure 9e. First Difference Log Tran.

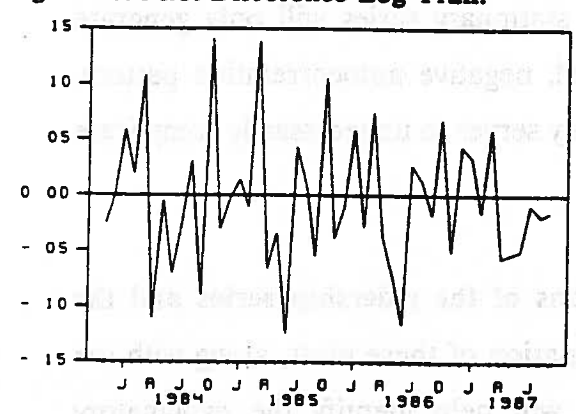


Figure 9f. Sample Autocorrelation Function

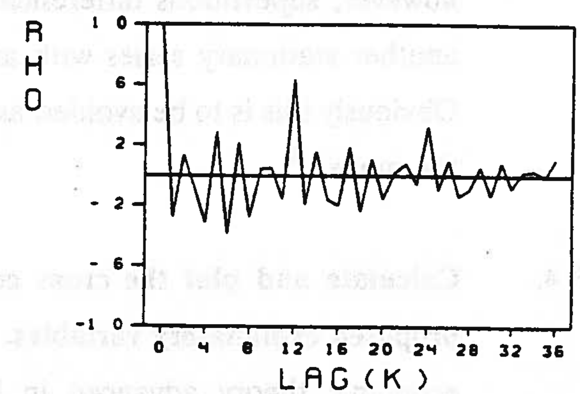


Figure 9g. First & Seas. Difference Log Tran.

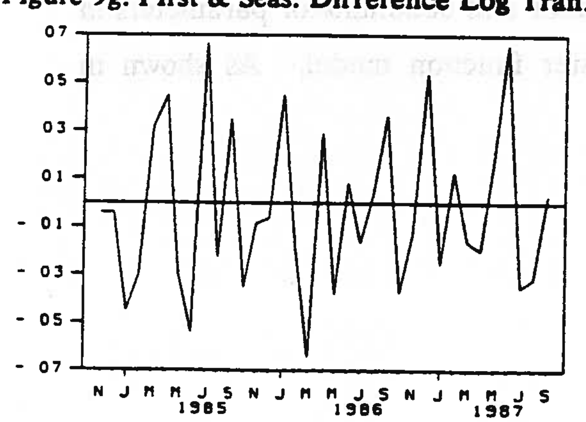


Figure 9h. Sample Autocorrelation Function

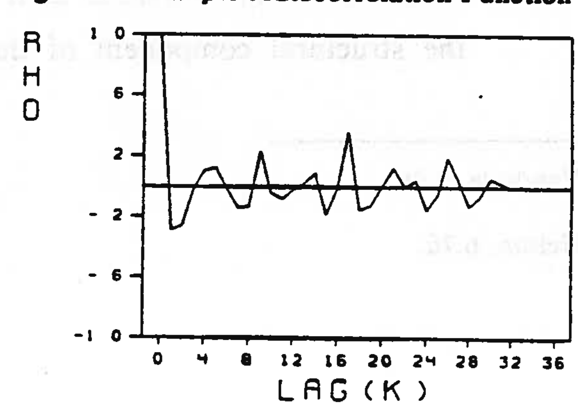
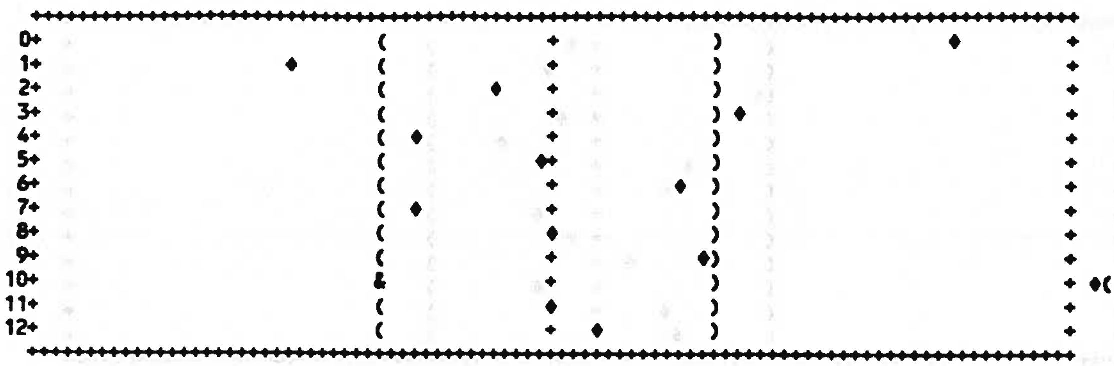
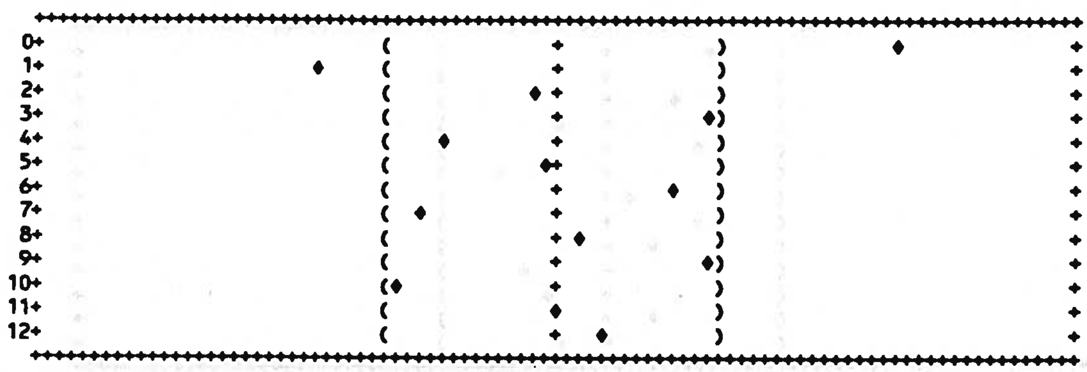


Figure 10a. Sample Cross Correlation Analysis

```
CROSS-CORRELATIONS OF SERIES   32     FUTRIPS  AND  34     FVMILES
NUMBER OF OBSERVATIONS    35
FROM     1984: 9 UNTIL     1987: 7
  0 :   .774542  -.497202  -.097364   .370329  -.249656  -.010121   .257544  -.254532   .021828
  9 :   .304339  -.333603   .013166   .102230
```




```
CROSS-CORRELATIONS OF SERIES   32     FUTRIPS  AND  33     FVHOURS
NUMBER OF OBSERVATIONS    35
FROM     1984: 9 UNTIL     1987: 7
  0 :   .667342  -.466590  -.043835   .295154  -.214621  -.003365   .233435  -.253428   .057177
  9 :   .299305  -.310048   .012008   .092212
```




```
CROSS-CORRELATIONS OF SERIES   32     FUTRIPS  AND  35     FAVFARE
NUMBER OF OBSERVATIONS    35
FROM     1984: 9 UNTIL     1987: 7
  0 :  -.646285  -.070599   .288499  -.346656   .146788   .143126  -.205918   .037381   .095041
  9 :  -.045486  -.109842   .005840   .047995
```

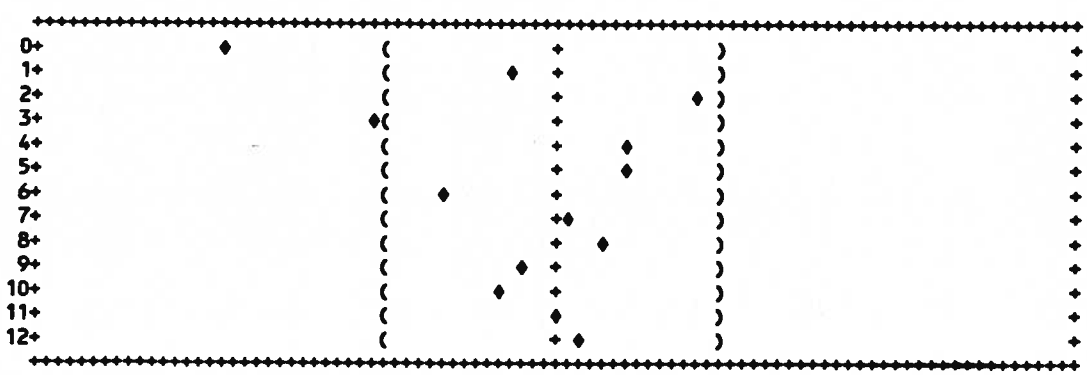
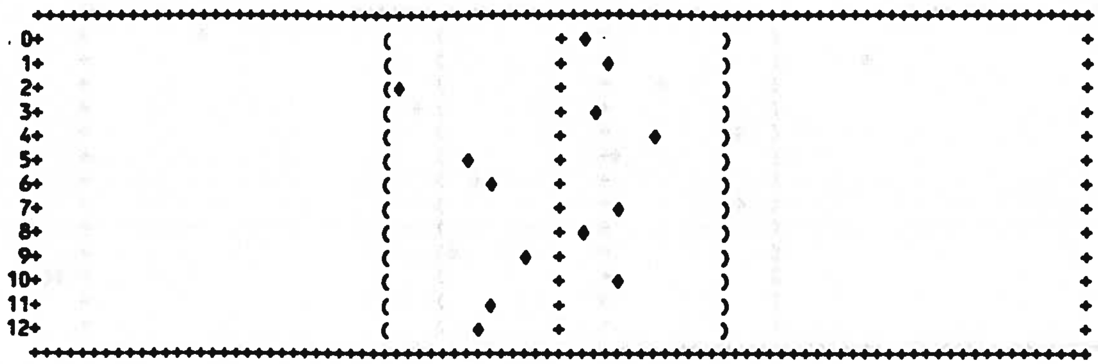


Figure 10b. Sample Cross Correlation Analysis (continued)

```
CROSS-CORRELATIONS OF SERIES   32      FUTRIPS  AND   38      FGAS
NUMBER OF OBSERVATIONS   35
FROM     1984: 9 UNTIL     1987: 7
  0 :     .052988   .091430  -.296225   .087173   .193760  -.176291  -.116628   .124833   .060538
  9 :    -.062428   .122625  -.120278  -.144297
```



```
CROSS-CORRELATIONS OF SERIES   32      FUTRIPS  AND   39      FEMPLO
NUMBER OF OBSERVATIONS   35
FROM     1984: 9 UNTIL     1987: 7
  0 :    -.183961   .005768  -.116706   .097423  -.162030   .204345  -.022474  -.164742  -.071784
  9 :     .159144   .126572  -.067822   .094216
```

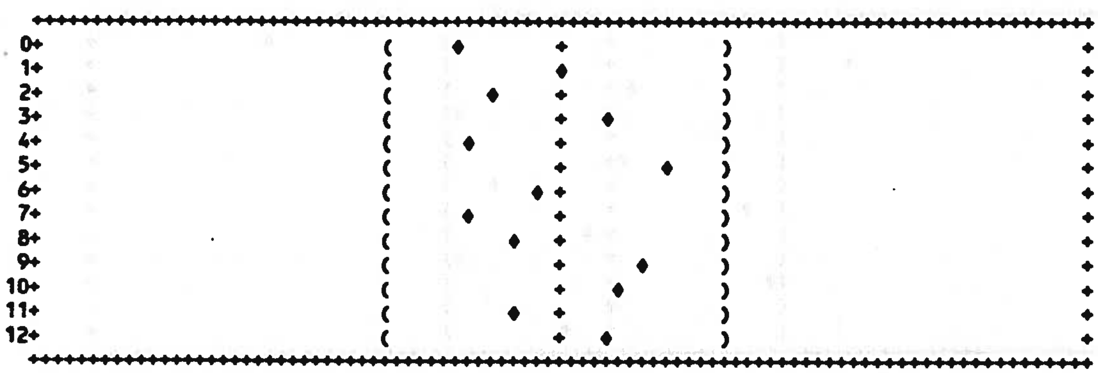


Figure 10a and Figure 10b, one should closely examine those observations whose cross correlations lie outside of the 95 percent large sample confidence interval.[71] The patterns found in the cross correlation plots are extremely helpful in identifying the dead time constant, which is equal to the lag of the first significant cross correlation, the denominator order (if any), which corresponds to the pattern of an AR(r) model and the numerator order, which is either the number of periods that the AR(r) pattern is delayed or, if there is no AR pattern, it corresponds to the order of the numerator parameters.[72]

STEP 5. **Identify and estimate the structural component of the transfer function.** Estimate the theoretically most viable structural model, which usually will contain service (SL), fare (FC), alternate mode costs (AC) and market characteristic (MC) variables. Include a lag structure sufficient to cover all lags suggested by economic theory and the cross correlation analysis. Re-estimate the equation, keeping only those variables and lags whose t-statistics are significant and whose coefficients are of the correct sign and magnitude. It is likely that alternate specifications, which the researcher may also wish to explore, will arise.

STEP 6. **Perform diagnostic checking on the equation.** Check each variable for its effect on the signs and magnitudes of the other coefficients, the standard error of the estimate (SEE), the corrected coefficient of determination, $\bar{R}^2$ and the covariance/correlation matrix (multicollinearity check). Remember that the constant should not be suppressed simply because of a low t-statistic.[73] In

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[71] In Figure 10a and Figure 10b, this confidence interval is represented by parentheses and is calculated by: $1.96 \times SE[r_{xy}(k)] = 1.96 \times 1/\sqrt{n}$ where n is the number of observations.

[72] See Vandaele, p.279.

[73] Cassidy, p.148-149.

building a transfer function model, it is very important to make sure that all factors explaining the dependent variable are represented in the model whenever possible. If the equation is not acceptable at this point, then return to Step 2 as additional explanatory variables are needed.

STEP 7. **Analyze the autocorrelation function (ACF) and the partial autocorrelation function (PACF) of the residuals a_t to determine the type of ARIMA model required.** A good technique is to plot both the ACF and the PACF and compare them with plots of known AR and MA functions. Our research indicates that the MA(1) × SMA(1) model provides a good fit for transit ridership.

STEP 8. **Estimate the transfer function with the ARIMA component and plot and analyze the new residuals.** The model is acceptable if the a_t residuals behave as white noise. In general, these conditions should be satisfied:

- The residual plot should show no systematic variation. In particular, a distinctive *sequence* of residuals is evidence that the noise model is inadequate.

- The residual autocorrelation function must show that the residuals are independent. The residual autocorrelation coefficients should lie within two standard errors, however a single point outside of this area is usually not evidence of autocorrelation.

- The Q statistic should show that the autocorrelation coefficients *as a group* are independently distributed. The Q statistic is approximately distributed as χ^2 with K-p-q degrees of freedom, where K is the number of autocorrelations in the sample and p and q are the respective orders of the AR and MA processes.[74]

[74] "Note the this chi-square test is a 'weak' hypothesis test. A value of Q below the 90 percent point on the chi-square distribution indicates that it is not necessary to accept the hypothesis that the residuals are non-white, since the probability that the hypothesis is true is less than 90 percent." From Robert Pindyck and Daniel L. Rubinfeld, *Econometric Models & Economic Forecasts*, p.550.

- The cross correlation functions between the explanatory variables and the residuals should show that the residuals are independent of each explanatory variable. The values of the individual cross correlations should lie within two standard errors. Again, one should be particularly wary of pronounced patterns.

If all the above conditions are met, then the model can be assumed to be in its final form.

As with all econometric techniques, **the researcher should not assume that the transfer function model can be applied in a cookbook fashion.** The reader is cautioned that the above algorithm is merely meant to serve as a guide to modeling transit ridership and that each situation will be unique in some way.

Fare Elasticity, Bus Ridership and Fare Revenues

A demand model always should be used to forecast ridership and revenue, but there are situations which necessitate a "rough and ready" approximation of patron reaction to fare change. Also, smaller transit properties may lack the means to build even a simple demand model. In these circumstances the fare elasticities presented here may serve as a guide to ridership forecasting.

There are four principal fare elasticity measures, point elasticity, shrinkage ratio, midpoint[75] elasticity and constant arc elasticity. The formulas for each are defined in equations 46-49 where F is fare and R is bus ridership.

$$\textit{Point elasticity} = \varepsilon_{pt} = \frac{\partial R}{\partial F} \cdot \frac{F}{R} \tag{46}$$

$$\textit{Shrinkage ratio} = \varepsilon_{sr} = \frac{\Delta R}{\Delta F} \cdot \frac{F}{R} = \frac{\Delta R/R}{\Delta F/F} \tag{47}$$

[75] Midpoint elasticity is sometimes referred to as arc elasticity.

$$\textit{Midpoint elasticity} = \varepsilon_{mid} = \frac{\Delta R}{\Delta F} \cdot \frac{F_2 + F_1}{R_2 + R_1} = \frac{(R_2 - R_1)(F_2 + F_1)}{(R_2 + R_1)(F_2 - F_1)} \tag{48}$$

$$\textit{Constant arc elasticity} = \varepsilon_{arc} = \frac{\ln R_2 - \ln R_1}{\ln F_2 - \ln F_1} \tag{49}$$

Mayworm and Lago[76] have shown that for fare changes less than 10 percent each formula yields about the same elasticity. However for measuring average elasticity at discrete values and over an extended range, the midpoint or constant arc formulas are the appropriate analytical tools. The general form of the demand functions estimated previously was that of a convex curve with a constant elasticity at all relevant points along the curve, therefore the constant arc formula is used here. Equation 49 can be algebraically manipulated to solve for new ridership, R_2 as shown in equation 50.

$$R_2 = \exp\left(\varepsilon_{arc} \times (\ln F_2 - \ln F_1) + \ln R_1\right) \tag{50}$$

It appears that equation 50 permits one to forecast ridership rather simply and with just four variables, existing fare, new fare, existing ridership and fare elasticity, but remember that the definition of fare elasticity includes the qualifier "assuming all other variables in the demand function remain constant." <u>This limitation is critical and must always be kept in mind when using any of the elasticity calculations to predict ridership</u>.

Table 11 and Table 12 were developed to help the transit planner apply the concepts advanced above. Both tables are based on the solution of equation 50 at several assumed fare increases and elasticities. Table 11 presents the estimated ridership loss and Table 12 presents the estimated revenue gain. The tables are self explanatory. For instance, assume that a transit system's estimated fare elasticity is -0.30 and the proposed fare increase is 50

[76] ***Patronage Impacts of Changes in Transit Fares and Services*, pp.4-8.**

Table 11. Ridership Loss at Several Assumed Fare Changes and Elasticities

Elast \ Fare	10%	20%	30%	40%	50%	60%
-0.10	0.9%	1.8%	2.6%	3.3%	4.0%	4.6%
-0.20	1.9%	3.6%	5.1%	6.5%	7.8%	9.0%
-0.30	2.8%	5.3%	7.6%	9.6%	11.5%	13.2%
-0.33	3.1%	5.8%	8.3%	10.5%	12.5%	14.4%
-0.40	3.7%	7.0%	10.0%	12.6%	15.0%	17.1%
-0.50	4.7%	8.7%	12.3%	15.5%	18.4%	20.9%
-0.60	5.6%	10.4%	14.6%	18.3%	21.6%	24.6%

Table 12. Fare Revenue Gain at Several Assumed Fare Changes and Elasticities

Elast \ Fare	10%	20%	30%	40%	50%	60%
-0.10	9.0%	17.8%	26.6%	35.4%	44.0%	52.7%
-0.20	7.9%	15.7%	23.4%	30.9%	38.3%	45.6%
-0.30	6.9%	13.6%	20.2%	26.6%	32.8%	39.0%
-0.33	6.6%	13.0%	19.2%	25.3%	31.2%	37.0%
-0.40	5.9%	11.6%	17.0%	22.4%	27.5%	32.6%
-0.50	4.9%	9.5%	14.0%	18.3%	22.5%	26.5%
-0.60	3.9%	7.6%	11.1%	14.4%	17.6%	20.7%

percent. Table 11 shows that ridership is expected to decrease 11.5 percent and fare revenues are expected to increase 32.8 percent. Notice that the all-hour average bus fare elasticity proposed earlier in this study, -0.40 is shaded.

APPENDIX A

REVIEW OF SELECTED FARE ELASTICITY STUDIES

1. Curtin, John F. "Effect of Fares on Transit Riding," *Highway Research Record*, 213 (1968), 8-20.

This is the most often quoted source of mass transit fare elasticities by one of the originators of the well-known Simpson and Curtin formula. To calculate fare elasticities, Curtin regressed percent net loss in passenger traffic against percent fare increase for 77 bus fare increases occuring over a 20 year period from 1947 to 1968. The result of these regressions is the following formula:

$$(\% \text{ net loss in passenger traffic}) = 0.80 + 0.30 \times (\% \text{ fare increase})$$

The equation shown above is known as the Simpson and Curtin formula. Although the data spanned a two decade period, the Simpson-Curtin study is a cross-sectional analysis, not a time series analysis. That is, the analysis was performed using data from a cross section of transit systems at different points in time over a twenty year period. Curtin does not explain what procedure, if any, he used to standardize the data to account for the time differences. Judging from this article only, Curtin did not attempt to deflate the fares, so estimates using the Simpson-Curtin formula should be performed in nominal dollars.

Besides quantifying a relationship between fare increases and ridership decreases, Curtin addressed two other very important points in this article. First, the absolute value of fare elasticity should approach unitary elasticity as fares increase past a certain level. Curtin states: "It is not until the minimum or basic fare level reaches 30 to 35 cents that diminishing returns may offset the revenue yield."[77] This threshold fare level is obviously in nominal dollars of the time (circa 1968) and, if it does in fact exist, defines the point where a fare increase would result in a decrease in total revenue.

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[77] P.14.

Secondly, Curtin realizes that service variables are also very important determinants of ridership. He states:

> There was an additional element to be considered in predicting the impact of fare changes on the BARTD and Muni rapid transit systems. For those particular facilities, a lower shrinkage loss was anticipated because of the superior quality of service and passenger amenities represented in rapid transit compared to existing forms of transit in the Bay Area. Patrons will be required to pay more than they do now, but they will be getting a faster and more comfortable journey in more attractive vehicles. While an increase in fares for rapid transit journeys above existing surface fares can be expected to have some effect on riding, it will be less that the passenger loss which would result from a higher fare on existing routes and service. In calculating the impact of a rate increase from the existing surface transit fare to a proposed higher rapid transit fare, therefore, a loss ratio of 0.10 was applied.[78]

In his actual ridership predictions, Curtin subjectively reduced the "Simpson-Curtin rule calculated" loss ratio by half in order to account for service variables. Curtin admits that the Simpson and Curtin formula does not directly address service factors and therefore the formula should be applied judiciously because of this shortcoming.

There is also a mathematical problem which develops when the Simpson and Curtin rule is applied to large fare increases. For example, suppose that a system increases its fares from \$0.25 to \$1.00, which is an admittedly large fare increase, but not completely unreasonable. The calculation of the Simpson and Curtin formula is:

$$(\%\text{ net loss in passenger traffic}) = 0.80 + 0.30 \times (\%\text{ fare increase})$$

$$(\%\text{ net loss in passenger traffic}) = 0.80 + 0.30 \times ((1.00-0.25)/0.25 \times 100)$$

$$(\%\text{ net loss in passenger traffic}) = 90.80$$

[78] P.14.

Obviously, a 90.80 percent loss in passenger traffic is impossible, if only because the captive ridership will remain in the system. The missing determinant of ridership loss here is the $1.00 fare in relation to the cost of alternate transportation modes. The point is that the Simpson and Curtin formula, like all fare elasticity measures, will provide erroneous results when used indiscriminately.

Lastly, there is some written evidence that a fare elasticity value identical to Simpson and Curtin's results had been in use since the beginning of the twentieth century. A search of the American Public Transportation Association's library files uncovered this memo:

> So-called Simpson formula assumes an overall average loss of three tenths of one percent in traffic for each one percent increase in average fare. It was not developed originally by Hawley Simpson as a formula but came out of certain data in a fare case in which he testified. However, similar formula of thumb was used in the in industry as early as 1917.[79]

2. Simmons, P.N., "No Quadratic Equations or Integral Calculus Required," *Mass Transportation*, May 1948.

P.N. Simmons attempts to quantify the relationship between fares and ridership by plotting passengers per vehicle mile against average fare using data collected from transit companies operating in the year 1946. Simmons then charted analogues of percent increase in average fare versus percent decrease in revenue passengers for six, seven, eight and nine cent fares and the weighted average of all fares, which was about 7.2 cents. These analogues permitted estimations of ridership loss if fares were raised. For instance, Simmons' analogues predicted a 15 percent ridership loss if fares are raised from six to seven cents. Using the shrinkage ratio calculation, this computes into a elasticity of -0.9, which is a very large fare elasticity value. Simmons points out that his elasticities are rather high and his subsequent analysis brings out an important point: the effect of fare increase

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[79] American Transit Association, internal memorandum, October 4, 1954.

on ridership depends on the size of the original fare in relation to the new fare. For example, Simmons' analogues predict a 15 percent ridership decrease for a six to seven cent increase and only a 5 percent ridership decrease for a nine to ten cent fare increase. The paper implies that fare elasticity varies depending upon the position on the demand curve that the market is at when the fare increase occurs.

3. Boyle, Daniel K. *Are Transit Riders Becoming Less Sensitive to Fare Increases?.* Washington: Transportation Research Board 64th Annual Meeting, January 1985.

In this paper, Boyle calculates fare elasticities from monthly fare and ridership data during the years 1979 to 1982. He then classifies these elasticities according to geographic region, SMSA size, year, original fare level, mode, fare increases and fare reductions. Boyle attempts to correct for the effects of seasonal and yearly trends by comparing the ridership of the month after the fare change with the ridership of the same month in the preceding year. He also calculates long term fare elasticities by comparing ridership levels in the sixth month following the fare change with the same month in the preceding year. Boyle's conclusions are:

> While increasing fares may have had little apparent impact on ridership in the energy conscious years of 1979 and 1980, this appears to have been only a temporary, and perhaps illusory, phenomenon.
>
> A second interesting point concerns the concept of a fare threshold. This concept postulates that, as fare rises beyond a certain threshold level, ridership behavior changes significantly. ... Elasticities are increasingly negative at higher levels of the original fare up to the "over 60 cent" category. In this category, ridership response becomes less elastic than in the "51 cent to 60 cent" category. The explanation driving this version would be that by the time a relatively high fare level is reached, most of the "choice" riders have already abandoned transit for another mode, and so further increases have less impact on ridership. While the data in Table 1 does not provide a conclusive proof that a fare threshold of this nature actually exists, further research into this concept would be useful.

> The conclusion that the Simpson and Curtin formula for measuring ridership response to fare changes has remained valid has significance beyond the scope of this study.[80]
>
> The hypothesis that transit ridership has become or is becoming less elastic with respect to fares must be rejected.[81]

4. Rainville, W.S. *Estimated Loss In Passenger Traffic Due to Increases in Fares (1961-1967).* Washington: American Transit Association, February 9, 1968.

   Rainville, W.S. *Estimated Loss in Passenger Traffic Incident to Increases in Urban Transit Fares.* New York: American Transit Association, November 24, 1961.

   Rainville, W.S. *Estimated Loss in Passenger Traffic Incident to Increases in Urban Transit Fares Since January 1, 1950.* New York: American Transit Association, November 24, 1961.

   Graves, Frank M., *Effect of Fare Changes on Ridership.* Washington: Institute of Public Administration, January 9, 1974.

Between 1947 and 1968, Walter Rainville of the American Transit Association wrote a series of papers investigating the effects of fare increases on ridership. In these papers, Rainville examined fare elasticities during a twenty year period from 1947 to 1967 using the "before and after" technique. The resulting elasticities were then grouped by population of principal city served. Rainville used a variation of the shrinkage ratio elasticity measure with which he attempted to normalize ridership by netting out the ridership percentage loss of the three months preceding the fare increase from the ridership percentage loss of the six months after the fare increase. His before and after methodology is paraphrased below:

> Average fare, under the previous rates, was based upon data for three months immediately prior to the increase in fares. Average fare, under new rates, was based upon data for a six-month period immediately after the

[80] P.6-7.

[81] P.i.

> increases. Months in which strikes or other unusual factors occurred were omitted. Fare increases generally have a varying effect on transit riding depending upon whether the increase becomes effective during the winter or summer months and for this reason it was considered advisable to take a longer period after the increase to fully compensate for the effect of the increased fares and avoid the possibility of distortion because of seasonal variations.
>
> The "Percent Change in Traffic Over the Previous Year" was also based upon the same three-month period prior to the increase and the same six-month period after the increase. The "Net Change in Traffic Trend" is the residual left after deducting "before increase trend of traffic" from the "after increase trend of traffic." The assumption made here is that the trend of traffic immediately preceding the fare increase represents a normal condition as compared with the accelerated decrease immediately following the change. The net difference between these two figures is taken as the actual percentage loss in traffic due to the fare increase.
>
> The "Percent Loss in Traffic for Each Percent Increase in Average Fare" is the ratio of the "Net Change in Traffic Trend" to the "Percent Increase in Average Fares."[82]

Given the number of factors which can and do affect ridership, Rainville's blanket assumption of ridership stability in the three months preceding the fare increase is certainly suspect. Also, Rainville's use of monthly average fare as a measure of price is also debatable. Average fare changes with the composition of the riders even when the official fare schedule remains unchanged. Therefore, a system offering deep student discounts will notice an increase in average fare during summer months, even though the price charged to passengers has remained unchanged. In these types of situations, which are quite common, it may be erroneous to assume that ridership is a direct function of average fare without some sort of seasonal calibration.

Rainville's elasticity measures are useful because of the large number of observations and the considerable time span his studies covered. His results are presented in Table 13.

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[82] *Estimated Loss In Passenger Traffic Due to Increases in Fares (1961-1967)*, p.1.

so that every elasticity estimate was comparable. This provided a base to generalize about transit passenger reaction to fare and service changes.

The authors also examined the methods with which fare and service elasticities have been estimated. They then grouped the elasticity studies into two categories: 1) quasi-experimental approaches and 2) non-experimental approaches. In outline form, the breakdown is:

1. quasi-experimental approaches - analyze actual fare and service changes
   a. estimating from demonstrations or practical experiments while controlling unmeasured influences
   b. monitoring actual changes in services or current fares
2. non-experimental approaches - do not analyze actual fare and service changes
   a. conventional time-series analysis of annual (not related to a specific fare or service change) transit operating statistics
   b. aggregate direct-demand and mode-split models based on cross-sectional data
   c. disaggregate behavioral mode-choice models based on cross-sectional data

The report strongly supports the quasi-experimental approaches and its warning against using elasticities derived from non-experimental models is given below.

> ... some problems are posed by reliance on elasticity estimates from a cross-sectional data base containing no fare or service changes. One can not rely on elasticity estimates from cross-sectional studies to provide accurate estimates of annual changes in patronage in response to fare and service changes, because the elasticity estimates from cross-sectional analysis reflect a different type of behavior from that of the annual change behavior implicit in time-series analysis. This difference between time-series and cross-sectional models arises because the residuals from both models cannot be assumed to belong to the same underlying population.
>
> The demand elasticities from the cross-sectional models are generally greater than the time-series estimates ... almost twice as large as elasticities estimated from a actual demonstrations and experiments. Although cross-sectional estimates have some advantages in forecasting structural changes in demand ... dynamic annual change type responses can not be

Table 13: Fare Elasticities from Rainville's Papers, 1947-1967

Population of Principal City Served	ELASTICITIES 1947-1952	1950-1961	1961-1967
More than 500,000	-0.34	-0.28	-0.22
100,000 to 500,000	-0.36	-0.33	-0.32
Less than 100,000	-0.33	-0.36	-0.43
TOTAL	-0.35	-0.32	-0.35

As shown in Table 13, all of Rainville's figures are very close to the Simpson-Curtin elasticity of -0.36. This is not surprising since both Curtin and Rainville, being contemporaries in time and field of study, probably used comparable databases. In any case, Rainville's calculations show a definite time trend toward inelasticity for the larger cities and toward elasticity for the smaller cities. In a related paper, Frank Graves explains: "The apparent decline in the first two rows of the table and the apparent increase in elasticities in the third row may be traced to the absolute value of the fares. That is, above 30 cents, there is probably a change in the elasticity or market response of riders."[83] In conclusion, Rainville's studies seem to support the theory that there may exist a fare threshold above which consumer behavior radically changes and fare elasticity declines.

5. Mayworm, Patrick D., Armando M. Lago and J. Matthew McEnroe. *Patronage Impacts of Changes in Transit Fares and Services*. Washington: U.S. Department of Transportation Urban Mass Transportation Administration, 1980.

This report is a comprehensive review of transit fare and service elasticity studies performed in the U.S. and the U.K. from 1947 to 1980. Although the authors did not undertake their own elasticity experiment, they did aggregate and present over one hundred different transit elasticities. The elasticities, which spanned time and space, were adjusted

[83] Frank Graves, *Effect of Fare Changes on Ridership*, p.1-2.

> estimated from them with any degree of confidence unless supporting time-series information is available... .
>
> This report shows that quasi-experimental approaches result in more stable elasticity estimates than the calibrated models relying on cross-sectional data, in spite of the alleged superiority of these models in controlling for the influence of exogenous variables.[84]

The authors suggest that logit/probit analysis, though statistically superior and more academically acceptable, will probably not bear results which are readily applicable to real-world situations.

Some of the major findings of the report are:[85]

1. Transit demand is inelastic to fare changes. Transit fare elasticities range in value from -0.04 to -0.87 with a mean of -0.28 ± 0.16 and are not appreciably different from the Simpson and Curtin formula.

2. Elasticities for fare increases (-0.34) do not differ from those for fare decreases (-0.37). [This is somewhat disturbing. It would seem to be very difficult to win back passengers with a fare decrease once they have left the system.]

3. Small cities (-0.35) have larger fare elasticities than large cities (-0.24). [A reason for this may be that smaller cities were developed during the automobile age and are more conducive to car travel.]

4. Off-peak fare elasticities (-0.40) are double the size of peak-fare elasticities (-0.17).

5. Short-distance trips (-0.55) are more elastic than long-distance trips (-0.29).

6. Fare elasticities rise with income and fall with age. [This supports the hypothesis that the "captive riders" are the poor and the elderly.]

7. Of all trip purposes, the work trip is the most inelastic (-0.10).

8. Ridership response to service changes is inelastic.

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[84] P.14-15.

[85] The following quotes are taken from pages ix-xvi.

9. Ridership is more responsive in lower-service areas.

10. Headway (-0.47) and vehicle-miles elasticities (+0.30 to +0.85) are similar.

11. Ridership is more responsive to improvement in headways (-0.42) than in-vehicle time (-0.29).

Chapter 2, "Elasticity of Demand and its Measurement" is an excellent summary of fare elasticity concepts and calculations. Two important points are stressed. First, "Without information on the demand curve, it is impossible to determine which demand elasticity measure best represents the transit market being observed" and second, ". . . whatever demand elasticity measure is used, one must be cautious about interpreting the results as definitive."[86]

Chapter 5 presents a step-by-step method of computing revenue impacts from fare and service changes, including dynamic interactions, by using the appropriate submarket elasticities. This algorithm, if properly applied, will allegedly furnish the transit planner with a good approximation of expected revenue changes.[87]

The most important hypothesis of this study is that the Simpson and Curtin rule is still valid for system-wide transit analysis, although riders in different cities and among the various transit submarkets within the same city may react differently to fare changes. Also, that the aggregate fare elasticity may not give a true indication of the impact on revenues from a fare increase. Therefore, a disaggregated demand model will be a more accurate predictor of revenues. The authors state: "Although the mean fare elasticity does not deviate much from the Simpson and Curtin rule of -0.30, there are several reasons to argue against the indiscriminate use of this aggregate value."[88] Lastly, this report also refutes

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[86] P.10.

[87] Pp. 84-88.

[88] P.82.

the notion that fares become more elastic at higher fare levels: "In summary, the theoretical agreement for suggesting that fare elasticity values are dependent on the size of the fare change and fare level have yet to be substantiated."[89]

6. Knudson, Bill and Michael A. Kemp. *The Effects of a 1976 Bus Fare Increase in Erie, Pennsylvania*, Working Paper 1428-01. Washington: The Urban Institute, 1980.

   Knudson, Bill and Michael A. Kemp. *The Effects of a 1976 Bus Fare Increase in the Kentucky Suburbs of Cincinnati*, Working Paper 1428-02. Washington: The Urban Institute, 1980.

   Knudson, Bill and Michael A. Kemp. *The Effects of a 1977 Bus Fare Increase in Forth Worth, Texas*, Working Paper 1428-04. Washington: The Urban Institute, 1980.

   Kemp, Michael A. *Planning for Fare Changes: A Guide to Developing, Interpreting, and Using Fare Elasticity Information for Transit Planners*, Working Paper 1428-05. Washington: The Urban Institute, 1980.

In this series of reports, Kemp and Knudson used monthly time series data and multiple linear regression analysis to estimate the interdependent effects of passenger ridership, bus service level and travel costs in relation to the respective supply and demand functions. The resulting simultaneous equations models were used to estimate ridership at the system-wide level and for the cash paying, token and pass paying, student, elderly and handicapped submarkets. The statistical methodology is advanced, employing two-stage least squares with first-order autocorrelation corrections, dummy variables for unknown data and an instrumental variable for the number of bus miles operated each month. Working Paper 1428-05 is a manual detailing how to use these econometric techniques to estimate fare and service elasticities. The other papers are dissertations of the fare elasticity studies of the respective cities.

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[89] P.22.

The general form of Kemp and Knudson's models is:

On the demand side, the number of bus trips made a month is influenced by, among other things, the fare and the level of service provided:

$$q = D(p, \mathbf{L}, ...)$$

where q is the number of bus trips made
p is the fare level; and
L is the vector of level of service attributes.

Typically, one does not have available any accurate time series data characterizing the various service attributes (speed, waiting time, comfort, and so on) which are believed or known to influence demand, nor to have information about the corresponding attributes of competitive modes, particularly the private automobile. The only readily available statistic which can be used as an imperfect proxy for the level of service is the number of bus miles operated, m. Thus it is common to examine variations in q over time as a function of p, m and any other relevant variables that are available.

But on the supply side, the amount of service provided in a given month is influenced by, inter alia, the transit management's expectation of what the ridership will be in that month. [. . .] The supply equation must be of the form:

$$m = S(\hat{q}, . . .)$$

where $\hat{q}$ is the transit management's expectation of q.

In econometric terms, the situation can be characterized as on in which q and m are jointly determined; attempts to estimate one of these equations without consideration of the other will lead to "simultaneous equations bias." The errors associated with measuring m are likely to vary systematically with levels of q, and estimation of the demand equation by ordinary least squares method will be inappropriate. One possible corrective measure is to devise an instrumental variable to be used in the stead of m, and that is the procedure chosen here.

. . . One common functional form frequently used in analyses of this type is the "constant elasticity" specification, in which the logarithm of the level of demand is regressed as a linear function of the logarithms of the price and (perhaps) of other influencing variables. As Appendix A details, this type of

> function has the property that the price elasticity is the coefficient of the "log price" variable, and is constant with respect to changes in the other influencing variables. However, in the case of transit ridership there is very little empirical evidence to support this "constant elasticity" hypothesis over anything other than a very small price change. To the contrary, most of the evidence suggest that fare elasticities do vary with service levels and other influencing factors.[90]

Kemp and Knudson used this technique to examine the impact of fare changes on ridership for the Transit Authority of Northern Kentucky (WP 1428-02), City Transit Services (WP 1428-04) and the Erie Metropolitan Transit Authority (WP 1428-01). His monthly database consisted of ridership counts by passenger/fare categories, vehicle miles, total population size and population of passenger categories, area-wide employment, gasoline price and availability, month length, weather factors and seasonal dummy variables. All monetary price variables are adjusted for inflation by using the general consumer price index to convert them to constant dollars. The number of observations were, Kentucky: 25 months, Erie: 48 months, Fort Worth: 27 months.

The statistical test results of these studies are generally very good with R^2 values of 0.9 to 0.99. The report stresses that model validation is imperative and that coefficients characteristics, as measured by their sign, magnitude and student-t test, must reasonably correlate with empirical evidence and theories. The authors state that elasticity estimation "cannot be prescribed beforehand to such a degree that it can be carried out in a semimindless way."[91]

The reports present both point and midpoint arc elasticity estimates and, in most cases, there is little difference between the point elasticity measured at the mean and the midpoint arc elasticity. The midpoint arc elasticity estimation method involves using the derived

[90] *Planning for Fare Changes*, p.31.

[91] Ibid., p.31.

Table 14. Fare Elasticities from Knudson/Kemp Papers

	Kentucky Suburbs	Erie, Pa.	Fort Worth, Texas
Point Elasticity	-0.12	-0.33	-0.38
Midpoint Arc Elasticity (3 months)	-0.08	-0.37	-0.44
Midpoint Arc Elasticity (6 months)	-0.11	-0.32	-0.46
Midpoint Arc Elasticity (9 months)	-0.12	-0.32	-0.39
Midpoint Arc Elasticity (1 year)	-0.12	-0.32	-0.41

equations to forecast "what the patronage would have been in the months following the fare change if in fact the fare had not been changed but if all other variables assumed the values observed for them in those months."[92] By using the forecasted ridership as the "before" case and the actual patronage as the "after" case, the arc elasticity can be measured. The aggregate system fare elasticities from the three areas studied are shown in Table 14.[93]

Note that the Erie and Fort Worth fare elasticities are close to the Simpson and Curtin formula, however the Kentucky suburbs elasticity is about one-third of the Simpson and Curtin estimation. This finding lends credence to the hypothesis that Simpson and Curtin, although generally accurate, may be grossly inadequate for a particular transit system or for a specific transit submarket.

The econometrics in these reports are good. The authors have worked to insure that the results are statistically adequate and empirically acceptable. In fact, they have adopted the bold position of stating in the abstract of one report that their study was partially inconclusive. In Working Paper 1428-04 they state: "However, in this case (Fort Worth) data limitations and statistical problems inhibited the ability to draw firm conclusions about

[92] Ibid., p.42.

[93] These elasticities shown in this table are from Bill Knudson and Michael A. Kemp, *The Effects of a 1976 Bus Fare Increase in the Kentucky Suburbs of Cincinnati*, p.43., *The Effects of a 1976 Bus Fare Increase in Erie, Pennsylvania*, p.41., *The Effects of a 1977 Bus Fare Increase in Fort Worth, Texas*, p.43.

fare elasticities for each of the market segments studied."[94] Also, they demonstrate the power of the two-stage least squares methodology to explicitly recognize the interdependence of bus service level and ridership. Most researchers incorrectly treat bus service as an exogenous variable which will lead to simultaneous equations bias.

The drawbacks of this type of analysis, as pointed out by the authors, center on violations of the basic assumptions of the multiple regression model.[95] The violations are:

1. presence of multicollinearity;
2. serial correlation of errors (autocorrelation); and,
3. inefficient coefficient estimates and resulting biased standard errors.

Three reasons why these violations are inherent to transit data are:

1. In transit, what happened last month has influence on what will happen this month;
2. Data unavailability causes the exclusion of important variables which change systematically, inevitably leading to serial correlation; and,
3. Data measurement errors are likely to be associated with time due to the same data collection methods and the people collecting the data.

Although statistical routines (covariance tests, Durbin-Watson test, Cochrane-Orcutt procedure, instrument variable estimation) can measure and partially correct for these violations, the least-squares derived coefficients may still not be the best linear unbiased estimators. In some cases market structure approximations, such as fare elasticity, will be

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[94] P.i.

[95] *Planning for Fare Changes*, p.29.

inefficient and inconsistent, even though the predictive qualities of the model are still accurate. In short, application of this sort of econometrics is something of an art.

Regarding future research, the authors make two very pertinent points:

> Because of this problem (violation of assumptions), it is sometimes argued that regression techniques should not be used for time series analysis. A class of stochastic models generically referred to as Box-Jenkins techniques has some advantages for forecasting purposes.[96]

and

> Remember again that the main reason for deriving elasticities is to obtain a summary measure of sensitivity for broad comparative purposes, and that for forecasting purposes one would wish to use the complete demand equation in any case.[97]

7. Kyte, Michael, James Stoner and Jonathan Cryer. *Development of Time-Series Based Transit Patronage Models*. 2 vols. Iowa City: The University of Iowa, 1985.

This report describes the development and application to the Tri-Met transit system (Portland, Oregon) of a time-series modeling technique generally known as the Box-Jenkins transfer function model. The transfer function methodology is an extension of the ARIMA (integrated autoregressive moving-average) technique. The basis of the ARIMA technique is the hypothesis that if a variable has exhibited some type of systematic behavior in the past one may mathematically model this behavior and use the model to infer something about probable future behavior. Very simply put, the ARIMA model is a kind of advanced extrapolation technique. The Box-Jenkins transfer function model provides a way to combine the ARIMA technique with regression analysis to "produce a better forecast than

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[96] Ibid., p.29.

[97] Ibid., p.54.

would be possible through the use of either of these techniques alone."[98] The idea is to use the information contained in the unexplained variance of the dependent variable to enhance the forecast of the structural model. This aspect makes the transfer function model a more accurate forecasting tool than regression analysis alone, especially when the structural relationships produced by regression analysis are themselves changing over time and when the regression residuals exhibit serial correlation in either a linear or cyclical fashion. The chief disadvantage to this type of analysis is that one must have some knowledge of both the causal relationships and their associated lag structures.

The transfer function model can "resolve several problems that occur when standard regression models are used with time series data."[99] The authors have outlined these problems, which are presented in Table 15.[100]

Table 15. Comparison of Standard Regression and Transfer Function Models

Comparison	Standard Regression	Transfer Function
1. Correlated input variables	Yes, the input variables are highly correlated. Multicollinearity is present.	No, data is differenced.
2. Autocorrelated errors	Yes, the error structure is highly autocorrelated, violating basic model assumptions.	Yes, but model structure allows for correlated errors.
3. Lag structure for input variables	No, only contemporaneous correlation is assumed	Yes, methodology directly investigates the nature of dynamic relationships.
4. Coefficient estimates and standard errors	Estimates are inefficient and the standard errors (and thus the significance tests) are biased.	Estimates are efficient and the standard errors are unbiased.

[98] Robert S. Pindyck and Daniel L. Rubinfeld, *Econometric Models & Economic Forecasts*, p.593.

[99] P.2.

[100] Table 4, p.15.

The reduction of the effects of some persistent regression problems makes time-series modeling a good econometric tool. Also, the transfer function model permits the transit planner to include past information about lagged relationships and seasonal ridership patterns into the forecast. However, one must assume that these lag structures and seasonal patterns are fairly consistent over time.

The authors used monthly and quarterly data from 1971 to 1982, which was provided directly by the Tri-Met system, to generate sixteen transit ridership models: one for the system as a whole, six representing distinct geographic sectors and nine for individual bus routes. The general form of the models is:

$$R_t = F(SL_t, TC_t, MS_t, S_t, I_t) + N_t$$

where R_t = transit ridership

SL_t = the level of transit service (route miles, platform miles or hours)

TC_t = travel costs by auto and by transit (fare, gasoline price)

MS_t = size of the travel market (population, employment of service area)

S_t = seasonal factors such as weather (temperature, rainfall, snowfall)

I_t = interventions (gasoline shortages, marketing plans, weather extremes)

N_t = the error structure

These structural relationships were then fitted to the transfer function.

At the system level, this study produced fare point elasticity of -0.29, a service level point elasticity of 0.51, a gasoline price point elasticity of 0.32 and a employment point elasticity of 0.49. Notice that the fare elasticity is rather close to the Simpson and Curtin formula. The system-wide predictive characteristics of the model were good with a mean absolute percent error of 2.1 percent for a 12 month forecast. The methodology used here is perhaps one of the better ways to forecast the effects of a fare increase, although the caliber of the causal relationships is probably no better than that of the standard regression model. This may be a consideration if it is deemed that determining market structure is of greater importance than providing an accurate forecasting tool.

At the route level, the study found that "the response delay to service level changes tends to be about two to three time longer for urban routes than for suburban routes."[101] At the geographic sector level, fare point elasticities varied from -0.13 to -0.42 with a mean value of -0.23. Route level fare elasticities varied from -0.35 to -0.90. Judging from these values, it seems that patrons of some routes are more sensitive to fares than the system as a whole. Taking this a step further, the routes whose fare elasticities approach unity may experience a drastic decrease in ridership if fares are raised. It is conceivable that across-the-board fare increases actually may reduce revenue on some Tri-Met routes.

The strongest objection to this model is that one gets the impression that the report, while mathematically correct, fails to apply basic economics principles. For instance, the authors state that "natural logarithms of the data were used, so that model coefficients give the elasticities directly for each variable."[102] It certainly makes analysis convenient when one may simply read the elasticities from the coefficients, however this is not the reason to use log transformations. Log transformations are sometimes used when one must fit a non-linear demand function with linear regression techniques. Log functions are also used in time-series analysis as a means of removing growth over time of the variance of the data. Many economists believe that "one should use transformations, of any type, only when one is reasonably sure that the demand curve is non-linear at the points of interest and this only because non-linear estimation is computationally expensive and laborious to validate since the normal significance tests are not directly applicable."[103]

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[101] P.5.

[102] P.5.

[103] Robert S. Pindyck and Daniel L. Rubinfeld, *Econometric Models & Economic Forecasts*, p.266.

The report is nebulous on the interpretation of the findings. The question: "how can this knowledge of elasticities be used to help Tri-Met achieve greater operational efficiency?" is not explicitly answered anywhere. Obviously, the aim of economic research should be to produce a complete and practical application.

The second shortcoming of this study is that it did not explicitly investigate the interdependence of ridership and service as the Kemp and Knudson studies did with their simultaneous equations models. The authors realized this and state:

> The transfer function models used here assume only a one-way dependence; that is, input variables affect the output variable, but not vice-versa. For example, as capacity limits are approached as ridership increases, additional service might be required. Thus ridership level will influence level of service. The general multiple time-series model developed by Tiao and Box (1981) has the ability to handle this two-way dependence.[104]

Finally, the study did not examine transit submarkets disaggregated by demographic characteristics, purpose of trip and peak versus off-peak fares. The data needed to explore these relationships may not have been available at the quarterly or monthly level. This type of investigation usually is left to the realm of the cross-sectional model. However, with the appropriate data proxies, the transfer function model may be a useful tool.

8. Batchelder, J.H., K.W. Forstall and J.A. Wensley. *Estimating Patronage for Community Transit Services*. Washington: U.S. Department of Transportation Urban Mass Transportation Administration, 1984.

Unlike the other papers reviewed, this report was designed to help transit planners predict ridership for new paratransit service, and therefore its findings are not readily applicable to fare elasticity research, which usually focuses on existing service where past

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[104] P.73.

operating data was available. It is included in this literature review because it provides a concise explanation of ridership estimation techniques, and it details a cross-sectional regression analysis method which purports to forecast ridership with only socio-economic data. Socio-economic factors greatly influence ridership in the long-run and a method which combines socio-economic effects with operating variables would be novel and useful. Much of the report concerns supply decisions including route layout and fare and service policy and these chapters are not reviewed here.

The report outlines five ridership estimation methods which are given below:[105]

1. Analogy – The use of ridership levels attained on similar services in similar locations. [The report presents some tables and analogues which chart weekday ridership as a function of population. One disadvantage of this method is that the distributions are quite dispersed, nonetheless, the information in the table can be used to obtain rough bounds on potential ridership.]

2. Elasticity – Service and fare elasticities derived from ridership and mode choice models may be used to estimate the impact of proposed supply changes. [The report discusses a technique to apply elasticities obtained from other research to project ridership within a certain confidence interval. Confidence intervals which are usually given in the 75 to 95 percent range are perhaps easier for non-technical people to understand than standard deviation.]

3. Direct Estimation – Ridership data from existing services have been synthesized into graphs, equations, nomographs, and similar techniques

[105] PP.18-19.

that can be applied to estimate ridership on a proposed service. [The use of nomographs here is very interesting and may be a good way for the researcher to convey his elasticities findings. The authors also present equations obtained by regressing average weekday ridership per square mile against population, area, fleet size, automobile ownership and percent of population over 64 years of age. This analysis is of limited use since fares are excluded, but the use of census data is appealing.]

4. Mode Choice – Ridership on proposed feeder services can be estimated using mode-choice models, provided that total travel from the served area to the line-haul route being fed can be measured or estimated. To use mode choice models, the user [also] needs information on traveler characteristics (such as auto availability) as well as on the service quality of optional modes and total travel.

5. Equilibrium Models – Direct estimation equations or mode-choice techniques can be imbedded in an analytical procedure that also estimates the service quality resulting from the estimated ridership. These supply and demand models are iteratively applied until their results converge. The advantage of an equilibrium model is that, unlike other models, it explicitly recognizes and responds to the high degree of interdependency between service quality and ridership. The principal disadvantage is the amount of data and time required for application.

The report addresses the fact that revenue estimation is difficult when complex fare structures are in place for different population groups.[106] As a solution to this problem, the authors used transit system and census data to calculate a propensity ratio by age group. This ratio is defined as the percent of all riders in the age group over the percent of all persons of that age group. The authors suggest that the researcher may wish to collect ridership data broken down by age group if ridership by fare category is unavailable. The propensity ratios should be transferable across cities, assuming that people in similar age brackets have similar transit needs.

Finally, the authors review mode choice models and attempt to adapt a specific model to general transit ridership. The mode choice models need so much data that they are infeasible for a nation-wide elasticity study, but these models illuminate an important point: it is the cost and service differences between bus and car travel that determine how people choose to travel. For instance, the model reviewed in this report uses bus versus car travel time differences and fares versus out-of-pocket auto expenses as casual variables. It is not the absolute level of bus service or cost which matters, but the level relative to competing modes.

The last chapter presents three case studies using the planning techniques which were evaluated in the earlier sections. The examples are set in Norfolk, Va., Merrill, Wis., and Baltimore, Md. These step-by-step procedures are the strongest part of this study as they lead the transit planner through some fairly complex calculations using actual situations. The appendices contain census tables that may be useful to future research, especially the income and auto ownership data.

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[106] P.62.

9. Rose, Geoffrey. "Transit Passenger Response: Short and Long Term Elasticities using Time Series Analysis," *Transportation*, 13,2 (1986), 131-141.

This study investigates the effects of gas price, fare, service and weather factors on ridership of the Chicago Transit Authority rail system. Since the study was limited to rail transit, it does not have direct implication on the current research. However, it was included in this literature review because the ARIMA model was used in the study and the issue of short vs. long term impacts was investigated.

The study used over 11 years of monthly data (135 observations) covering the period from January 1970 to March 1981. During this period, there was a downward trend in ridership (1970-78) followed by an upward trend. The cash fare increased in four steps from 40 cents to 80 cents. The period of study included the energy crisis in 1974 and a substantial gas price increase in 1979-80. The data base is shown in Table 16.

Table 16. Variables and Descriptions in CTA Rail Ridership Model

Variable	Description (Units)
(A) Dependent variable ridership	Average weekday ridership on the CTA rail system (unlinked passenger trips)
(B) Explanatory variables service	Weekday service (train miles)
Fare	(i) Adult cash fare (cents) (referred to as "Cash Fare") (ii) "Deflated Fare" (March 1981 cents) - "Cash Fare" deflated by Monthly Consumer Price Index
Cost of car trips	(i) Deflated gas price (March 1981 ¢/gal) (ii) Cost per mile of car trips (March 1981 ¢/mile) - deflated gas price divided by average fleet miles per gallon
Climatic effects	(i) Average daily rainfall (ii) Average daily snowfall

The final equation is as follows:

$$R_t = -153852.0 + 0.56R_{t-1} + 0.44R_{t-12} - 0.24R_{t-13} + 1001.7GAS_{t-1} - 1782.4GAS_{t-12}$$
$$+ 1168.2GAS_{t-13} + 11.3SER_{t-12} - 5.78SER_{t-13} - 989.8FARE_{t-1} + 2896FARE_{t-12}$$
$$- 1474.7FARE_{t-13} + 184SNOW_{t-12} + a_t$$

where R_t is ridership at time t, GAS_t is deflated gas price at time t, SER_t is service at time t, $FARE_t$ is deflated fare at time t, $SNOW_t$ is average daily snowfall at time t, and a_t is an error term.

Tests show that the coefficients of FARE and SNOW variables are not statistically significant. The implications of the equation are:

- Short term gas price elasticity is 0.11
- Short term service elasticity is zero
- Short term fare elasticity is zero

The demand equation is then rewritten to move all the ridership terms to the left hand side of the equation. The cumulative effect over time of each explanatory variable in the equation constitutes a long term effect. The results:

- Long term gas price elasticity is 0.18
- Long term service elasticity is 1.84
- Long term fare elasticity is zero

Geoffrey Rose concludes:

> The service reduction would presumably cut costs. Since our model predicts a twelve month delay before service changes influence ridership, we would not expect the service change to have any short term effect on revenue. The fare increase should result in both short and long term revenue increases since a zero fare elasticity is indicated by the model. With improved revenue and reduced costs the new policy should improve the operator's financial position, at least in the short term.

> In the long term the service reductions will lead to significant ridership losses because the long term service elasticity is greater than 1. This ridership loss will tend to offset the revenue increase obtained from the higher fares.[107]

While the Rose study is interesting in that it separates the short term (one month) and long term (one year) effects of changes in gas price and service elasticities, it is not likely to be accepted by many economists and researchers. The study was not based on a solid economic theory. The notion that fare does not affect rail ridership, either in the short or long term, even for the marginal "choice" riders, is difficult to accept. In addition, the conclusion that the reduction of service (train miles) will not result in an immediate effect on ridership, which implies that there is little or no substitution for rail transit in Chicago, needs further verification.

[107] P.142.

APPENDIX B

SURVEY QUESTIONNAIRE FORM

American Public Transit Association
1201 New York Avenue, NW - Suite 400
Washington, DC 20005
May 1988

Return by June 10, 1988
Contact: Jim Linsalata
(202) 898-4125

Transit System: ______________________________

Mailing Address: ______________________________

Prepared by: ______________________ Phone (____)_____-_____ ext.____

INSTRUCTIONS

This survey requests that you provide *monthly* data for some very basic fixed-route motor bus statistics. By collecting and compiling this information, APTA will attempt to provide all participating transit systems with a comprehensive, quantitative study on how passengers respond to fare increases. This research project will help transit planners gauge and prepare for the effects of future fare changes on motor bus ridership. Although the completion and return of this form to APTA is entirely voluntary, we hope that you will fill out all the items to the best of your ability. If you have any questions, please do not hesitate to call Jim Linsalata, Manager - Research and Statistics at (202) 898-4125.

There are three parts to this survey:

- Part I: TOTAL MOTOR BUS (page 3)
- Part II: MOTOR BUS - Peak Periods (page 4)
- Part III: SIGNIFICANT EVENTS affecting ridership (page 5)

In Part II, the peak period data are for A.M. and P.M. peaks combined and "Peak Periods" are as defined by your system. We realize that not all systems are capable of separating out the peak period operating statistics. In this case, fill out Parts I and III only. In Part III, we ask that you report any significant events which may have caused an unusual motor bus ridership growth or decline.

On page 2 are definitions of the survey forms' column headings. Hopefully, these definitions will unambiguously identify the data items that you have been asked to supply. Also included in these definitions are the **corresponding Section 15 account numbers**, which may be conducive to those transit systems which report to UMTA.

Lastly, please provide your best estimate whenever a month(s') data is missing and insert an asterisk (*) next to the estimated figure(s). A sound analysis of your transit system can not be conducted unless there is a continuous sequence of comparable data. In most cases, a knowledgeable guess is very close to what the actual number is and entirely adequate for the purposes of this study. These raw statistics will not be published and are for the internal use of APTA's Research and Statistics Department only. APTA will only publish the mathematical analyses and equations and the general conclusions and recommendations of the project.

*** Thanks for your time and effort ***

Fixed-Route Motor Bus	Transit service provided by all motor bus type operations along regularly scheduled routes. Excludes all types of demand response service.
Base Adult Cash Fare	The amount of fare paid for a single ride, excluding zone and transfer charges, during the off-peak period by passengers who are not entitled to reduced fares and who pay the fare with money.
Peak Adult Cash Fare	The amount of fare paid for a single ride, excluding zone and transfer charges, during the peak periods by passengers who are not entitled to reduced fares and who pay the fare with money.
Unlinked Passenger Trips	Transit trips taken by both initial-board (originating) and transfer (continuing) transit patrons. Each passenger is counted each time that person boards a transit vehicle regardless of the type of fare paid or transfer presented. Corresponds to UMTA Section 15, Form Number 406, line 12.
Vehicle Revenue Miles	Sum of all passenger vehicle miles operated while in revenue service. Excludes miles travelled to and from storage facilities and other dead-head travel. Corresponds to UMTA Section 15, Form Number 406, line 4.
Revenue Vehicle Hours	Total number of hours revenue vehicles are operated in revenue service. Excludes hours consumed while travelling to and from storage facilities and during other dead-head travel. Corresponds to UMTA Section 15, Form Number 406, line 6.
Passenger Fare Revenues For Transit Service	Revenue earned from carrying passengers along regularly scheduled routes. Includes the base fare, zone premiums, extra cost transfers and quantity purchase discounts applicable to the passenger's ride. Also included is "park and ride" revenue. Excluded are charter service revenues, school bus service revenues and all non-transportation revenues. Corresponds to UMTA Section 15, Form Numbers 201 or 202, Account Number 401.

Date	Base Adult Cash Fare	Unlinked Passenger Trips	Vehicle Revenue Miles	Vehicle Revenue Hours	Passenger Fare Revenues For Transit Service
Jan 1984	$0.65				
Feb	•				
Mar	•				
Apr	•				
May	•				
Jun	•				
Jul	•				
Aug	•				
Sep	•				
Oct	•				
Nov	•				
Dec	•				
Jan 1985	•				
Feb	•				
Mar	•				
Apr	•				
May	•				
Jun	•				
Jul	•				
Aug	•				
Sep	•				
Oct	•				
Nov	•				
Dec	•				
Jan 1986	$0.85				
Feb	•				
Mar	•				
Apr	•				
May	•				
Jun	•				
Jul	•				
Aug	•				
Sep	•				
Oct	•				
Nov	•				
Dec	•				
Jan 1987	•				
Feb	•				
Mar	•				
Apr	•				
May	•				
Jun	•				
Jul	•				
Aug	•				
Sep	•				
Oct	•				
Nov	•				
Dec	•				

Date	Peak Adult Cash Fare	Unlinked Passenger Trips	Vehicle Revenue Miles	Vehicle Revenue Hours	Passenger Fare Revenues For Transit Service
Jan 1984	$0.65				
Feb	•				
Mar	•				
Apr	•				
May	•				
Jun	•				
Jul	•				
Aug	•				
Sep	•				
Oct	•				
Nov	•				
Dec	•				
Jan 1985	•				
Feb	•				
Mar	•				
Apr	•				
May	•				
Jun	•				
Jul	•				
Aug	•				
Sep	•				
Oct	•				
Nov	•				
Dec	•				
Jan 1986	$0.85				
Feb	•				
Mar	•				
Apr	•				
May	•				
Jun	•				
Jul	•				
Aug	•				
Sep	•				
Oct	•				
Nov	•				
Dec	•				
Jan 1987	•				
Feb	•				
Mar	•				
Apr	•				
May	•				
Jun	•				
Jul	•				
Aug	•				
Sep	•				
Oct	•				
Nov	•				
Dec	•				

Transit System: ______________________________

1. Your morning peak period hours are from __________ to __________ a.m.

2. Your evening peak period hours are from __________ to __________ p.m.

3. Please check each passenger category (one check every line) to describe the types of unlinked passenger trips reported in Part I and Part II.

	All	Some	None
Adult	____	____	____
Reduced	____	____	____
Transfer	____	____	____
Charter	____	____	____
Free	____	____	____

4. Please report all work stoppages from ____________ until ____________

Date began ________________ Date ended ________________

Date began ________________ Date ended ________________

Date began ________________ Date ended ________________

5. Please list any other events and their dates which have had a significant impact on your bus ridership. Include such occurrences as special marketing promotions, major bus route extensions or a worker lay off by a principal employer.

6. If requested, would your transit system be able to supply the data in Parts I and II by:

Fare category (i.e. elderly, student) ____ Not at all ____ Completely
____ Partially: ______________________

Bus route (i.e. suburban, intercity) ____ Not at all ____ Completely
____ Partially: ______________________

APPENDIX C

EQUATIONS AND VARIABLES

The equations which have been described earlier in this report are presented in this appendix in standard regression output format. The output is ordered alphabetically according to principal city served by the transit agency. Below is a list of the notation used in the regression output.

List of Notations Used in Regression Output

Ridership Variable

LUTRIPS	natural log of unlinked trips per months
DUTRIPS	first regular difference of LUTRIPS
FUTRIPS	first regular and seasonal difference of LUTRIPS

Service Variables

LVMILES	natural log of revenue vehicle miles per month
DVMILES	first regular difference of LVMILES
N_DVMILE	specifies that DVMILES is a numerator coefficient in the polynomial
D_DVMILE	specifies that DVMILES is a denominator coefficient in the polynomial
FVMILES	first regular and seasonal difference of LVMILES
N_FVMILE	specifies that FVMILES is a numerator coefficient in the polynomial
D_FVMILE	specifies that FVMILES is a denominator coefficient in the polynomial
LVHOURS	natural log of revenue vehicle hours per month
DVHOURS	first regular difference of LVHOURS
N_DVHOUR	specifies that DVHOURS is a numerator coefficient in the polynomial
D_DVHOUR	specifies that DVHOURS is a denominator coefficient in the polynomial
FVHOURS	first regular and seasonal difference of LVHOURS
N_FVHOUR	specifies that FVHOURS is a numerator coefficient in the polynomial
D_FVHOUR	specifies that FVHOURS is a denominator coefficient in the polynomial
LWORKDAY	natural log of number of working days in the month
DWORKDAY	first regular difference of LWORKDAY
N_DWORKD	specifies that DWORKDAY is a numerator coefficient in the polynomial

D_DWORKD	specifies that DWORKDAY is a denominator coefficient in the polynomial
FWORKDAY	first regular and seasonal difference of LWORKDAY
N_FWORKD	specifies that FWORKDAY is a numerator coefficient in the polynomial
D_FWORKD	specifies that FWORKDAY is a denominator coefficient in the polynomial

Fare Variables

LAVFARE	natural log of real average fare per month
DAVFARE	first regular difference of LAVFARE
N_DAVFAR	specifies that DAVFARE is a numerator coefficient in the polynomial
D_DAVFAR	specifies that DAVFARE is a denominator coefficient in the polynomial
FAVFARE	first regular and seasonal difference of LAVFARE
N_FAVFAR	specifies that FAVFARE is a numerator coefficient in the polynomial
D_FAVFAR	specifies that FAVFARE is a denominator coefficient in the polynomial
LBSFARE	natural log of real base fare
DBSFARE	first regular difference of LBSFARE
N_DBSFAR	specifies that DBSFARE is a numerator coefficient in the polynomial
D_DBSFAR	specifies that DBSFARE is a denominator coefficient in the polynomial
FBSFARE	first regular and seasonal difference of LBSFARE
N_FBSFAR	specifies that FBSFARE is a numerator coefficient in the polynomial
D_FBSFAR	specifies that FBSFARE is a denominator coefficient in the polynomial

Alternate Mode Cost Variables

LGAS	natural log of real local monthly unleaded gasoline price in cents per gallon
DGAS	first regular difference of LGAS
N_DGAS	specifies that DGAS is a numerator coefficient in the polynomial
D_DGAS	specifies that DGAS is a denominator coefficient in the polynomial
FGAS	first regular and seasonal difference of LGAS
N_FGAS	specifies that FGAS is a numerator coefficient in the polynomial
D_FGAS	specifies that FGAS is a denominator coefficient in the polynomial

Local Travel Market Variables

LEMPLO	natural log of local employment
DEMPLO	first regular difference of LEMPLO
N_DEMPLO	specifies that DEMPLO is a numerator coefficient in the polynomial

D_DEMPLO	**specifies that DEMPLO is a denominator coefficient in the polynomial**
FEMPLO	**first regular and seasonal difference of LEMPLO**
N_FEMPLO	**specifies that FEMPLO is a numerator coefficient in the polynomial**
D_FEMPLO	**specifies that FEMPLO is a denominator coefficient in the polynomial**

Error Structure

MA	**the coefficient for the regular moving average polynomial, θ**
MA_SEAS	**the coefficient for the seasonal moving average polynomial, Θ**
AR	**the coefficient for the regular autoregressive polynomial, ϕ**
AR_SEAS	**the coefficient for the seasonal autoregressive polynomial, Φ**
RHO	**the coefficient for first-order serial correlation correction, ρ**

Intervention Variables

The type and form of the intervention variables are defined at the far right side of the equation output, if used.

```
CAPITAL DISTRICT TRANSIT AUTHORITY, ALBANY, NY -- SYSTEM TOTAL  Tue  09-12-1989

CONVERGENCE REACHED ON ITERATION     25
DEPENDENT VARIABLE   35      FUTRIPS
FROM     1984: 6 UNTIL     1987: 3
TOTAL OBSERVATIONS      34       SKIPPED/MISSING          0
USABLE OBSERVATIONS     34       DEGREES OF FREEDOM      28
R**2               .63358927     RBAR**2          .56815878
SSR           .41008796E-01      SEE           .38270091E-01
DURBIN-WATSON  1.93937693
Q( 15)=    15.5931          SIGNIFICANCE LEVEL  .409598
NO.     LABEL     VAR  LAG    COEFFICIENT    STAND. ERROR     T-STATISTIC    SIGNIF LEVEL
***    *******    ***  ***   ************    ************    ************    ************
  1    CONSTANT    1    0    .1300530E-03    .1174240E-02     .1107551       .9126007
  2    N_FAVFAR    2    1   -.4556279        .1333739        -3.416170       .1959040E-02
  3    N_FWORKD    3    0    .4255098        .1288596         3.302119       .2626187E-02
  4    N_FEMPLO    4    0    .8034025        .5201686         1.544504       .1336946
  5    MA          5    1   -.9546064        .1196703        -7.976973       .1094079E-07
  6    MA_SEAS     6   12   -.4917656        .2450001        -2.007206       .5446859E-01
```

```
ALEXANDRIA TRANSIT COMPANY, ALEXANDRIA, VA -- SYSTEM TOTAL  Mon  08-14-1989

CONVERGENCE REACHED ON ITERATION      2
DEPENDENT VARIABLE   32      FUTRIPS
FROM     1985:12 UNTIL     1988: 4
TOTAL OBSERVATIONS      29       SKIPPED/MISSING          0
USABLE OBSERVATIONS     29       DEGREES OF FREEDOM      24
R**2               .92223722     RBAR**2          .90927676
SSR           .59383376E-02      SEE           .15729931E-01
DURBIN-WATSON  2.03070561
Q( 14)=    11.0472          SIGNIFICANCE LEVEL  .682322
NO.     LABEL     VAR  LAG    COEFFICIENT    STAND. ERROR     T-STATISTIC    SIGNIF LEVEL
***    *******    ***  ***   ************    ************    ************    ************
  1    CONSTANT    1    0   -.9020895E-02    .2925083E-02    -3.083979       .5078913E-02
  2    N_FVMILE    2    0    .9961782        .6445766E-01    15.45477        .5659476E-13
  3    N_FAVFAR    3    0   -.1356247        .5931556E-01    -2.286494       .3134567E-01
  4    N_FAVFAR    4    2   -.2764988        .6418428E-01    -4.307889       .2414119E-03
  5    N_FEMPLO    5    5    .7286597        .3469121         2.100415       .4638169E-01
```

```
LEHIGH & NORTHAMPTON TRANSP AUTH, ALLENTOWN PA -SYSTEM TOTAL  Wed  09-20-1989

CONVERGENCE REACHED ON ITERATION      2
DEPENDENT VARIABLE   33      FUTRIPS
FROM     1986: 7 UNTIL     1988: 4
TOTAL OBSERVATIONS      22       SKIPPED/MISSING          0
USABLE OBSERVATIONS     22       DEGREES OF FREEDOM      18
R**2               .74016627     RBAR**2          .69686065
SSR           .77598606E-01      SEE           .65658462E-01
DURBIN-WATSON  2.22868938
Q( 11)=    5.56778          SIGNIFICANCE LEVEL  .900593
NO.     LABEL     VAR  LAG    COEFFICIENT    STAND. ERROR     T-STATISTIC    SIGNIF LEVEL
***    *******    ***  ***   ************    ************    ************    ************
  1    CONSTANT    1    0   -.3222015E-02    .1452492E-01    -.2218266       .8269461
  2    N_FVMILE    2    0    .9595755        .3322448         2.888158       .9792567E-02
  3    N_FAVFAR    3    0   -.7466957        .2866821        -2.604612       .1792469E-01
  4    N_FGAS      4    2    .6180107        .1808205         3.417814       .3068017E-02
```

VALLEY TRANSIT, APPLETON, WI -- SYSTEM TOTAL Thu 09-21-1989

CONVERGENCE REACHED ON ITERATION 37
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1986: 6 UNTIL 1988: 5
TOTAL OBSERVATIONS 24 SKIPPED/MISSING 0
USABLE OBSERVATIONS 24 DEGREES OF FREEDOM 19
R**2 .67914718 RBAR**2 .61159922
SSR .46071483E-01 SEE .49242410E-01
DURBIN-WATSON 1.99128713
Q(12)= 8.66210 SIGNIFICANCE LEVEL .731488

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	.2803322E-02	.6367744E-02	.4402379	.6647322
2	N_FVMILE	2	0	.5953103	.1561584	3.812221	.1176496E-02
3	N_FAVFAR	3	0	-.2550770	.8931534E-01	-2.855914	.1011054E-01
4	N_FEMPLO	4	3	2.769706	1.604273	1.726455	.1004899
5	AR	5	1	-.5815990	.2128860	-2.731974	.1324208E-01

ATLANTA RAPID TRANSIT AUTH., ATLANTA, GA -- SYSTEM TOTAL Thu 09-21-1989

DEPENDENT VARIABLE 15 LUTRIPS
FROM 1985:10 UNTIL 1988: 5
TOTAL OBSERVATIONS 32 SKIPPED/MISSING 0
USABLE OBSERVATIONS 32 DEGREES OF FREEDOM 28
R**2 .55973374 RBAR**2 .51256236
SSR .48771339E-01 SEE .41735279E-01
DURBIN-WATSON 1.98093443
Q(15)= 10.0778 SIGNIFICANCE LEVEL .814818

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	0	0	-4.284700	3.656719	-1.171734	.2511784
2	LVMILES	17	0	1.308261	.2456031	5.326731	.1135891E-04
3	LAVFARE	19	0	-.2773415	.1021073	-2.716176	.1118999E-01
4	LGAS	21	4	.1234886	.6114380E-01	2.019642	.5308414E-01

MASS TRANSIT ADMINISTRATION, BALTIMORE, MD -- SYSTEM TOTAL Thu 09-21-1989

CONVERGENCE REACHED ON ITERATION 12
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1986: 1 UNTIL 1988: 4
TOTAL OBSERVATIONS 28 SKIPPED/MISSING 0
USABLE OBSERVATIONS 28 DEGREES OF FREEDOM 23
R**2 .81029931 RBAR**2 .77730788
SSR .15865527E-01 SEE .26264149E-01
DURBIN-WATSON 2.15233871
Q(14)= 9.33534 SIGNIFICANCE LEVEL .808987

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	.2001158E-02	.2451575E-02	.8162744	.4227173
2	N_FVMILE	2	0	.9932840	.1386751	7.162671	.2744487E-06
3	N_FAVFAR	3	0	-.4945899	.1454712	-3.399917	.2459021E-02
4	AR	4	1	-.4241379	.1946146	-2.179373	.3979918E-01
5	MA_SEAS	5	12	-.5994563	.2498388	-2.399372	.2491892E-01

BROOME COUNTY DOT, BINGHAMTON, NEW YORK -- SYSTEM TOTAL Mon 08-28-1989
Note: Fare decrease 2/87 and peak period fare increase 1/88.

DEPENDENT VARIABLE 14 LUTRIPS
FROM 1985: 7 UNTIL 1988: 4
TOTAL OBSERVATIONS 34 SKIPPED/MISSING 0
USABLE OBSERVATIONS 34 DEGREES OF FREEDOM 28
R**2 .94447161 RBAR**2 .93455582
SSR .29727244E-01 SEE .32583543E-01
DURBIN-WATSON 1.62694046
Q(15)= 12.7076 SIGNIFICANCE LEVEL .624871

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	0	0	-8.316528	3.358539	-2.476234	.1958890E-01
2	LVMILES	16	0	.8564042	.9631365E-01	8.891826	.1203634E-08
3	LAVFARE	18	0	-.7040280	.6428706E-01	-10.95132	.1247666E-10
4	LEMPLO	21	4	1.799060	.7349262	2.447946	.2089388E-01
5	LGAS	20	3	.2267076	.1557147	1.455917	.1565376

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
6	RHO	1	0	.9006541	.8385748E-01	10.74030	.1944032E-10

APPALCART, BOONE, NC -- SYSTEM TOTAL Wed 10-04-1989

DEPENDENT VARIABLE 15 LUTRIPS
FROM 1985:10 UNTIL 1988: 5
TOTAL OBSERVATIONS 32 SKIPPED/MISSING 0
USABLE OBSERVATIONS 32 DEGREES OF FREEDOM 27
R**2 .83741116 RBAR**2 .81332392
SSR 1.5908830 SEE .24273771
DURBIN-WATSON 1.62683846
Q(15)= 11.5711 SIGNIFICANCE LEVEL .711160

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL	
1	CONSTANT	0	0	-19.46635	10.29761	-1.890376	.6948641E-01	
2	LVMILES	17	0	.3560326	.1842909	1.931906	.6392821E-01	
3	LAVFARE	19	0	-.5283814	.9319885E-01	-5.669398	.5080957E-05	
4	SKITRIPS	10	0	.7473281	.1718372	4.349047	.1748237E-03	ski bus 12/85-2/86=1
5	LEMPLO	22	3	5.385468	2.465714	2.184141	.3781264E-01	

NIAGARA FRONTIER TRANS AUTH, BUFFALO, NY -- SYSTEM TOTAL Thu 09-21-1989

CONVERGENCE REACHED ON ITERATION 25
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1984:12 UNTIL 1987:10
TOTAL OBSERVATIONS 35 SKIPPED/MISSING 0
USABLE OBSERVATIONS 35 DEGREES OF FREEDOM 30
R**2 .85954578 RBAR**2 .84081855
SSR .44027453E-01 SEE .38309030E-01
DURBIN-WATSON 2.08799195
Q(15)= 3.90400 SIGNIFICANCE LEVEL .998035

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	-.2383917E-02	.1076806E-02	-2.213877	.3458213E-01
2	N_FVMILE	2	0	1.040585	.2037086	5.108203	.1716263E-04
3	N_FAVFAR	3	0	-.5025439	.1539108	-3.265164	.2736278E-02
4	MA	4	1	-.9052168	.1054444	-8.584776	.1408601E-08
5	MA_SEAS	5	12	-.8205350	.2237953	-3.666453	.9463851E-03

CHATTANOOGA AREA TRANSP AUTH, CHATTANOOGA, TN - SYSTEM TOTAL Thu 09-28-1989

DEPENDENT VARIABLE 14 LUTRIPS
FROM 1983:12 UNTIL 1987: 6
TOTAL OBSERVATIONS 43 SKIPPED/MISSING 0
USABLE OBSERVATIONS 43 DEGREES OF FREEDOM 37
R**2 .89513740 RBAR**2 .88096678
SSR .10268268 SEE .52680239E-01
DURBIN-WATSON 1.89808143
Q(18)= 13.4187 SIGNIFICANCE LEVEL .766123

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL	
***	*******	***	***	************	************	************	************	
1	CONSTANT	0	0	-.6857713	1.585005	-.4326620	.6677718	
2	LAVFARE	18	0	-.3410170	.7181561E-01	-4.748509	.3060170E-04	
3	LVMILES	16	0	.9083531	.1293874	7.020416	.3034561E-07	
4	LGAS	20	4	.4381379	.1192262	3.674847	.7496773E-03	
5	JUNE	43	0	.6379124E-01	.2573040E-01	2.479217	.1784841E-01	peak dummy, June=1

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
6	RHO	1	0	.5441899	.1430860	3.803236	.5179842E-03

SW OHIO REGIONAL TRAN. AUTH., CINCINNATI, OH -SYSTEM TOTAL Thu 09-28-1989
Note that only peak fare increased in the observation period.

CONVERGENCE REACHED ON ITERATION 24
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1984: 7 UNTIL 1987: 2
TOTAL OBSERVATIONS 32 SKIPPED/MISSING 0
USABLE OBSERVATIONS 32 DEGREES OF FREEDOM 26
R**2 .82820808 RBAR**2 .79517117
SSR .11425709E-01 SEE .20963071E-01
DURBIN-WATSON 1.86480309
Q(15)= 10.0491 SIGNIFICANCE LEVEL .816642

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	N_FVHOUR	1	0	1.009202	.1120714	9.004999	.1793654E-08
2	N_FAVFAR	2	0	-.4724744	.1643895	-2.874116	.7971295E-02
3	N_FAVFAR	3	3	-.2655963	.1339684	-1.982529	.5808013E-01
4	N_FEMPLO	4	0	1.638314	.7230811	2.265741	.3202627E-01
5	MA	5	1	-.6483990	.2442322	-2.654847	.1336399E-01
6	MA_SEAS	6	12	-.6914339	.2245109	-3.079734	.4845499E-02

PINELLAS SUNCOAST, CLEARWATER, FL -- SYSTEM TOTAL Tue 08-29-1989

CONVERGENCE REACHED ON ITERATION 10
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1985: 3 UNTIL 1987: 9
TOTAL OBSERVATIONS 31 SKIPPED/MISSING 0
USABLE OBSERVATIONS 31 DEGREES OF FREEDOM 25
R**2 .78118648 RBAR**2 .73742378
SSR .82551512E-02 SEE .18171573E-01
DURBIN-WATSON 1.82740271
Q(15)= 17.0833 SIGNIFICANCE LEVEL .313908

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	N_FVMILE	1	0	1.199072	.1082947	11.07231	.3758481E-08
2	N_FAVFAR	2	1	-.2524697	.6463147E-01	-3.906297	.6300093E-03
3	N_FAVFAR	3	3	-.2253775	.7062809E-01	-3.191046	.3798378E-02
4	N_FEMPLO	4	1	1.283381	.6022256	2.131064	.4309936E-01
5	N_FGAS	5	0	.1151262	.6161567E-01	1.868456	.7346105E-01
6	AR	6	1	.4335103	.2045317	2.119526	.4415139E-01

DALLAS AREA RAPID TRANSIT, DALLAS, TX -- SYSTEM TOTAL Fri 09-29-1989

DEPENDENT VARIABLE 15 LUTRIPS
FROM 1985:10 UNTIL 1987: 9
TOTAL OBSERVATIONS 24 SKIPPED/MISSING 0
USABLE OBSERVATIONS 24 DEGREES OF FREEDOM 21
R**2 .91664881 RBAR**2 .90871060
SSR .19776617E-01 SEE .30687842E-01
DURBIN-WATSON 1.76122261
Q(12)= 18.4361 SIGNIFICANCE LEVEL .103077

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	0	0	3.911968	1.473740	2.654450	.1483086E-01
2	LVHOURS	16	0	.9299271	.1343944	6.919388	.7802270E-06
3	LAVFARE	19	0	-.1341959	.7565728E-01	-1.773734	.9061038E-01

REGIONAL TRANSPORTATION DISTRICT, DENVER, CO -- SYSTEM TOTAL Thu 08-10-1989

CONVERGENCE REACHED ON ITERATION 18
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1986:12 UNTIL 1988: 4
TOTAL OBSERVATIONS 17 SKIPPED/MISSING 0
USABLE OBSERVATIONS 17 DEGREES OF FREEDOM 10
R**2 .95233384 RBAR**2 .92373414
SSR .23514020E-02 SEE .15334282E-01
DURBIN-WATSON 2.31116701
Q(8)= 2.48777 SIGNIFICANCE LEVEL .962299

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	.2180658E-02	.5101370E-03	4.274652	.1625034E-02
2	N_FVMILE	2	0	.1862532	.2677556E-01	6.956088	.3918134E-04
3	N_FAVFAR	3	0	-.5618114	.2727859E-01	-20.59532	.1610742E-08
4	N_FGAS	4	3	.1771221	.1770949E-01	10.00154	.1587313E-05
5	AR	5	1	-.8658785	.2988350E-01	-28.97513	.5588236E-10
6	AR	6	2	-.7853894	.1899856E-01	-41.33941	.1643784E-11
7	MA_SEAS	7	12	-12.01261	6.089663	-1.972623	.7680888E-01

NORTRAN, DES PLAINES, IL -- SYSTEM TOTAL Mon 10-02-1989

CONVERGENCE REACHED ON ITERATION 18
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1985: 6 UNTIL 1988: 1
TOTAL OBSERVATIONS 32 SKIPPED/MISSING 0
USABLE OBSERVATIONS 32 DEGREES OF FREEDOM 27
R**2 .76061387 RBAR**2 .72514925
SSR .22155900E-01 SEE .28645922E-01
DURBIN-WATSON 1.80871028
Q(15)= 10.8629 SIGNIFICANCE LEVEL .762255

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	N_FVMILE	1	0	.2393053	.1090961	2.193528	.3705877E-01
2	N_FAVFAR	2	3	-.1167303	.6682030E-01	-1.746928	.9201772E-01
3	N_FEMPLO	3	0	1.084957	.5669890	1.913542	.6633629E-01
4	N_FEMPLO	4	1	1.170514	.5127809	2.282678	.3054598E-01
5	MA	5	1	-1.147786	.7394538E-01	-15.52208	.3718994E-08

SEMTA, DETROIT, MI -- SYSTEM TOTAL Mon 10-02-1989

DEPENDENT VARIABLE 16 LUTRIPS
FROM 1981: 7 UNTIL 1985: 6
TOTAL OBSERVATIONS 48 SKIPPED/MISSING 0
USABLE OBSERVATIONS 48 DEGREES OF FREEDOM 44
R**2 .92639149 RBAR**2 .92137273
SSR .85407805E-01 SEE .44057763E-01
DURBIN-WATSON 1.48478249
Q(18)= 41.0290 SIGNIFICANCE LEVEL .150842E-02

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL	
1	CONSTANT	0	0	4.895947	.5837010	8.387765	.1126889E-09	
2	LVMILES	18	0	.6269748	.4746616E-01	13.20888	.1984491E-16	
3	LAVFARE	20	0	-.2471996	.7768990E-01	-3.181875	.2683628E-02	
4	STRIKE	11	0	-.2182493	.4913385E-01	-4.441934	.5937241E-04	strike 12/83=1

SUN METRO, EL PASO, TX -- SYSTEM TOTAL Mon 10-02-1989

CONVERGENCE REACHED ON ITERATION 10
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1986: 2 UNTIL 1988: 4
TOTAL OBSERVATIONS 27 SKIPPED/MISSING 0
USABLE OBSERVATIONS 27 DEGREES OF FREEDOM 21
R**2 .60004315 RBAR**2 .50481533
SSR .68219635E-01 SEE .56996088E-01
DURBIN-WATSON 1.99459749
Q(13)= 9.54586 SIGNIFICANCE LEVEL .730582

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	.3192132E-02	.7571543E-02	.4215960	.6776042
2	N_FVMILE	2	0	.7667410	.2435115	3.148685	.4846655E-02
3	N_FAVFAR	3	0	-.2941322	.1157016	-2.542162	.1896411E-01
4	N_FEMPLO	4	3	2.484201	1.484348	1.673597	.1090415
5	N_FGAS	5	2	.2078270	.1312295	1.583691	.1282087
6	AR	6	1	-.4814393	.2151293	-2.237907	.3619362E-01

LANE TRANSIT DISTRICT, EUGENE, OR -- SYSTEM TOTAL Tue 08-22-1989
NOTE: Two fare increases occured in the observation period

CONVERGENCE REACHED ON ITERATION 14
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1986:11 UNTIL 1988: 3
TOTAL OBSERVATIONS 17 SKIPPED/MISSING 0
USABLE OBSERVATIONS 17 DEGREES OF FREEDOM 9
R**2 .90990714 RBAR**2 .83983491
SSR .14854358E-02 SEE .12847117E-01
DURBIN-WATSON 2.16076592
Q(8)= 6.51759 SIGNIFICANCE LEVEL .589457

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL	
1	CONSTANT	1	0	-.1329802E-01	.2891664E-02	-4.598744	.1292837E-02	
2	N_FVHOUR	2	0	.5849499	.9045601E-01	6.466678	.1158661E-03	
3	N_FVHOUR	3	1	.2000057	.9732660E-01	2.054995	.7004534E-01	
4	N_FAVFAR	4	0	-.1844477	.9782900E-01	-1.885409	.9200837E-01	
5	N_FEMPLO	5	0	2.983207	.5522466	5.401948	.4319084E-03	
6	N_FGAS	6	3	.2679025	.5916571E-01	4.528003	.1430290E-02	
7	N_FREEFA	7	0	.4856280E-01	.1652384E-01	2.938953	.1651482E-01	free fare, August=1
8	AR	8	1	-.6359163	.2380369	-2.671503	.2555942E-01	

CITY OF EVERETT TRANSIT, EVERETT, WA -- SYSTEM TOTAL Tue 10-03-1989

CONVERGENCE REACHED ON ITERATION 17
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1982: 9 UNTIL 1985: 6
TOTAL OBSERVATIONS 34 SKIPPED/MISSING 0
USABLE OBSERVATIONS 34 DEGREES OF FREEDOM 29
R**2 .56747991 RBAR**2 .50782197
SSR 1.1803392 SEE .20174576
DURBIN-WATSON 2.24720249
Q(15)= 16.0365 SIGNIFICANCE LEVEL .379627

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	.2812078E-02	.1489032E-01	.1888527	.8515237
2	N_FVMILE	2	0	1.167583	.4774416	2.445500	.2077148E-01
3	N_FAVFAR	3	0	-.4292831	.2361292	-1.818001	.7940852E-01
4	AR	4	1	-.3585123	.1526158	-2.349117	.2584150E-01
5	MA_SEAS	5	12	-.7384232	.2695802	-2.739159	.1042368E-01

MASS TRANSPORTATION AUTHORITY, FLINT, MI -- SYSTEM TOTAL Tue 10-03-1989

DEPENDENT VARIABLE 14 LUTRIPS
FROM 1984: 7 UNTIL 1988: 1
TOTAL OBSERVATIONS 43 SKIPPED/MISSING 0
USABLE OBSERVATIONS 43 DEGREES OF FREEDOM 37
R**2 .88466633 RBAR**2 .86908070
SSR .99311472E-01 SEE .51808241E-01
DURBIN-WATSON 1.59164647
Q(18)= 32.1316 SIGNIFICANCE LEVEL .212115E-01

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	0	0	-.3387878	2.798530	-.1210592	.9042992
2	LVMILES	16	0	.6179540	.1081416	5.714304	.1534373E-05
3	LAVFARE	18	0	-.5853808	.1963050	-2.981996	.5042832E-02
4	LEMPLO	21	0	.6018511	.4422106	1.361006	.1817443
5	LGAS	20	4	.3141586	.1476329	2.127972	.4006554E-01

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
6	RHO	1	0	.6406330	.1301138	4.923636	.1785985E-04

FRESNO TRANSIT, FRESNO, CA -- SYSTEM TOTAL Mon 08-14-1989

CONVERGENCE REACHED ON ITERATION 10
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1985: 2 UNTIL 1987: 6
TOTAL OBSERVATIONS 29 SKIPPED/MISSING 0
USABLE OBSERVATIONS 29 DEGREES OF FREEDOM 23
R**2 .78847797 RBAR**2 .74249492
SSR .18768547E-01 SEE .28566130E-01
DURBIN-WATSON 2.00856254
Q(14)= 10.0978 SIGNIFICANCE LEVEL .754996

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	.1270011E-02	.2004065E-02	.6337175	.5325174
2	N_FVMILE	2	0	.6313597	.1637661	3.855253	.8053527E-03
3	N_FAVFAR	3	0	-.3114969	.6239728E-01	-4.992155	.4756309E-04
4	N_FEMPLO	4	4	1.470020	.4270108	3.442582	.2217192E-02
5	AR	5	1	-1.059056	.1549733	-6.833795	.5761889E-06
6	AR	6	2	-.7687962	.1557530	-4.935995	.5467286E-04

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FORT WAYNE PUBLIC TRANSPORTATION CORPORATION -- SYSTEM TOTAL  Tue  08-22-1989
NOTE that is this is a fare decrease

DEPENDENT VARIABLE    14      LUTRIPS
FROM      1985: 6 UNTIL       1987:12
TOTAL OBSERVATIONS      31       SKIPPED/MISSING            0
USABLE OBSERVATIONS     31       DEGREES OF FREEDOM        28
R**2               .90270800     RBAR**2            .89575857
SSR            .53547358E-01     SEE            .43731060E-01
DURBIN-WATSON   1.80537975
Q(  15)=    15.3213          SIGNIFICANCE LEVEL  .428532
NO.      LABEL      VAR  LAG    COEFFICIENT    STAND. ERROR    T-STATISTIC    SIGNIF LEVEL
***     *******     ***  ***   ************    ************    ************   ************
 1      CONSTANT     0    0    5.316005        .7115395        7.471131       .3883927E-07
 2      LVMILES     16    0    .5517875        .6645144E-01    8.303620       .4912728E-08
 3      LAVFARE     18    5   -.1160577        .6538481E-01   -1.774994       .8677219E-01
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GRAND RAPIDS AREA TRANSIT AUTH, GRAND RAPIDS, MI -- SYSTEM TOTAL  Tue  08-29-1989

CONVERGENCE REACHED ON ITERATION      9
DEPENDENT VARIABLE    32      FUTRIPS
FROM      1985: 1 UNTIL       1987: 7
TOTAL OBSERVATIONS      31       SKIPPED/MISSING            0
USABLE OBSERVATIONS     31       DEGREES OF FREEDOM        26
R**2               .86195724     RBAR**2            .84071989
SSR            .36240849E-01     SEE            .37334686E-01
DURBIN-WATSON   1.98512019
Q(  15)=    13.0426          SIGNIFICANCE LEVEL  .599008
NO.      LABEL      VAR  LAG    COEFFICIENT    STAND. ERROR    T-STATISTIC    SIGNIF LEVEL
***     *******     ***  ***   ************    ************    ************   ************
 1      CONSTANT     1    0   -.4258731E-03    .4548501E-02   -.9362933E-01   .9261214
 2      N_FVMILE     2    0    .7659336        .1359957        5.632042       .6393642E-05
 3      N_FAVFAR     3    0   -.4304945        .6244050E-01   -6.894475       .2553956E-06
 4      N_FGAS       4    3    .1049703        .6983031E-01    1.503220       .1448328
 5      AR           5    1   -.4853606        .1769196       -2.743397       .1086762E-01
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GRAND RAPIDS AREA TRANSIT AUTH, GRAND RAPIDS, MI -- PEAK PERIOD  Tue  08-29-1989

CONVERGENCE REACHED ON ITERATION     10
DEPENDENT VARIABLE    32      FUTRIPS
FROM      1985: 1 UNTIL       1987: 7
TOTAL OBSERVATIONS      31       SKIPPED/MISSING            0
USABLE OBSERVATIONS     31       DEGREES OF FREEDOM        26
R**2               .82945482     RBAR**2            .80321711
SSR            .27343875E-01     SEE            .32429732E-01
DURBIN-WATSON   1.86958807
Q(  15)=    11.6091          SIGNIFICANCE LEVEL  .708341
NO.      LABEL      VAR  LAG    COEFFICIENT    STAND. ERROR    T-STATISTIC    SIGNIF LEVEL
***     *******     ***  ***   ************    ************    ************   ************
 1      CONSTANT     1    0    .9235601E-03    .4263173E-02    .2166368       .8301823
 2      N_FVMILE     2    0    .8447388        .1319465        6.402129       .8797003E-06
 3      N_FAVFAR     3    0   -.1910926        .6324671E-01   -3.021384       .5587348E-02
 4      N_FGAS       4    3    .1041211        .6195152E-01    1.680687       .1048004
 5      AR           5    1   -.3780691        .1871842       -2.019770       .5382457E-01
```

GRAND RAPIDS AREA TRANSIT AUTH, GRAND RAPIDS, MI -- OFF-PEAK PERIOD Tue 08-29-1989

CONVERGENCE REACHED ON ITERATION 6
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1984:10 UNTIL 1987: 7
TOTAL OBSERVATIONS 34 SKIPPED/MISSING 0
USABLE OBSERVATIONS 34 DEGREES OF FREEDOM 30
R**2 .80958656 RBAR**2 .79054521
SSR .10441150 SEE .58994773E-01
DURBIN-WATSON 2.18908522
Q(15)= 9.50645 SIGNIFICANCE LEVEL .849585

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	-.1637045E-02	.6724772E-02	-.2434350	.8093254
2	N_FVMILE	2	0	.7962177	.1646208	4.836676	.3693567E-04
3	N_FAVFAR	3	0	-.5331329	.6837867E-01	-7.796772	.1063324E-07
4	AR	4	1	-.5100564	.1651837	-3.087812	.4316334E-02

WESTSIDE TRANSIT LINES, GRETNA, LA -- SYSTEM TOTAL Thu 08-10-1989

CONVERGENCE REACHED ON ITERATION 11
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1986: 5 UNTIL 1988: 5
TOTAL OBSERVATIONS 25 SKIPPED/MISSING 0
USABLE OBSERVATIONS 25 DEGREES OF FREEDOM 20
R**2 .80523260 RBAR**2 .76627912
SSR .27779166E-01 SEE .37268731E-01
DURBIN-WATSON 1.83658030
Q(12)= 12.4830 SIGNIFICANCE LEVEL .407712

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	-.1924301E-03	.5631177E-02	-.3417227E-01	.9730785
2	N_FVMILE	2	0	.8511188	.1692615	5.028425	.6438542E-04
3	N_FAVFAR	3	0	-.3535910	.1142110	-3.095946	.5697006E-02
4	N_FEMPLO	4	1	1.811981	1.101910	1.644399	.1157213
5	AR	5	1	-.3543234	.2325491	-1.523650	.1432529

HONOLULU DOT SERVICES, HONOLULU, HI -- SYSTEM TOTAL Tue 10-03-1989

CONVERGENCE REACHED ON ITERATION 24
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1983: 7 UNTIL 1986: 5
TOTAL OBSERVATIONS 35 SKIPPED/MISSING 0
USABLE OBSERVATIONS 35 DEGREES OF FREEDOM 30
R**2 .81975018 RBAR**2 .79571687
SSR .47078709E-02 SEE .12527132E-01
DURBIN-WATSON 1.93489623
Q(15)= 11.2228 SIGNIFICANCE LEVEL .736648

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	-.5042844E-03	.1773415E-02	-.2843579	.7780898
2	N_FVMILE	2	0	.7230039	.8257083E-01	8.756166	.9171895E-09
3	N_FAVFAR	3	0	-.6520074	.1088353	-5.990773	.1430708E-05
4	MA_SEAS	4	12	-.5114261	.2490503	-2.053505	.4883053E-01
5	MA_SEAS	5	24	.7662790	.4417924	1.734478	.9309905E-01

KANSAS CITY AREA TRAN. AUTH, KANSAS CITY, MO -- SYSTEM TOTAL Tue 10-03-1989

```
DEPENDENT VARIABLE     14      LUTRIPS
FROM     1981: 4 UNTIL     1984:12
TOTAL OBSERVATIONS      45      SKIPPED/MISSING          0
USABLE OBSERVATIONS     45      DEGREES OF FREEDOM      40
R**2              .93032295     RBAR**2          .92335525
SSR            .80816853E-01    SEE           .44949097E-01
DURBIN-WATSON  1.92617375
Q( 18)=    13.9430          SIGNIFICANCE LEVEL  .732799
```

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	0	0	7.523322	1.772390	4.244732	.1263293E-03
2	LVMILES	15	0	.3071097	.1424719	2.155580	.3718793E-01
3	LAVFARE	17	0	-.5110148	.1182209	-4.322540	.9943390E-04
4	LGAS	19	2	.4264952	.1476030	2.889475	.6204225E-02

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
5	RHO	1	0	.4019595	.1502974	2.674427	.1078898E-01

RED ROSE TRANSIT AUTHORITY, LANCASTER, PA -- SYSTEM TOTAL Thu 08-17-1989

```
CONVERGENCE REACHED ON ITERATION     10
DEPENDENT VARIABLE     32      FUTRIPS
FROM     1984: 9 UNTIL     1987: 6
TOTAL OBSERVATIONS      34      SKIPPED/MISSING          0
USABLE OBSERVATIONS     34      DEGREES OF FREEDOM      30
R**2              .81249161     RBAR**2          .79374077
SSR            .23096793E-01    SEE           .27746947E-01
DURBIN-WATSON  1.74721056
Q( 15)=     8.26274         SIGNIFICANCE LEVEL .912817
```

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	N_FVMILE	1	0	.6755340	.1348118	5.010939	.2258809E-04
2	N_FAVFAR	2	0	-.4276713	.1453370	-2.942617	.6223506E-02
3	N_FEMPLO	3	1	1.124706	.7663964	1.467526	.1526397
4	MA	4	1	-.6408853	.1676768	-3.822147	.6210257E-03

LINCOLN TRANSPORTATION SYSTEM, LINCOLN, NE -- SYSTEM TOTAL Wed 10-04-1989

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CONVERGENCE REACHED ON ITERATION     15
DEPENDENT VARIABLE     32      FUTRIPS
FROM     1984: 2 UNTIL     1986: 8
TOTAL OBSERVATIONS      31      SKIPPED/MISSING          0
USABLE OBSERVATIONS     31      DEGREES OF FREEDOM      26
R**2              .61008044     RBAR**2          .55009281
SSR             .10513504       SEE           .63589741E-01
DURBIN-WATSON  2.24209281
Q( 15)=     9.87357         SIGNIFICANCE LEVEL .827616
```

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	-.7789331E-02	.6617492E-02	-1.177082	.2498264
2	N_FVMILE	2	3	.6238929	.2001503	3.117122	.4420729E-02
3	N_FAVFAR	3	0	-.5000035	.1532535	-3.262592	.3083597E-02
4	N_FGAS	4	3	.2480285	.1330360	1.864371	.7359769E-01
5	AR	5	1	-.7504399	.1769400	-4.241211	.2486388E-03

SOUTHERN CALIFORNIA RAPID TRAN DIST, LOS ANGELES, CA -- SYSTEM TOTAL Mon 09-11-1989

DEPENDENT VARIABLE 14 LUTRIPS
FROM 1983: 8 UNTIL 1987: 6
TOTAL OBSERVATIONS 47 SKIPPED/MISSING 0
USABLE OBSERVATIONS 47 DEGREES OF FREEDOM 43
R**2 .87676214 RBAR**2 .86816415
SSR .25737967E-01 SEE .24465432E-01
DURBIN-WATSON 2.05625536
Q(18)= 19.3316 SIGNIFICANCE LEVEL .371675

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	0	0	3.659720	1.192244	3.069606	.3705104E-02
2	LVMILES	16	0	.8395873	.7575562E-01	11.08284	.3719023E-08
3	LAVFARE	18	0	-.2307494	.3958631E-01	-5.829022	.6516533E-06

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
4	RHO	1	0	.8179273	.7221222E-01	11.32672	.3719005E-08

SOUTHERN CALIFORNIA RAPID TRAN DIST, LOS ANGELES, CA -- PEAK PERIOD Mon 09-11-1989

CONVERGENCE REACHED ON ITERATION 8
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1984: 8 UNTIL 1987: 6
TOTAL OBSERVATIONS 35 SKIPPED/MISSING 0
USABLE OBSERVATIONS 35 DEGREES OF FREEDOM 31
R**2 .93047161 RBAR**2 .92374305
SSR .15381300E-01 SEE .22274896E-01
DURBIN-WATSON 2.00469104
Q(15)= 18.6408 SIGNIFICANCE LEVEL .230474

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	-.5744157E-02	.2510352E-02	-2.288187	.2910325E-01
2	N_FVMILE	2	0	.9963581	.4596117E-01	21.67826	.3718988E-08
3	N_FAVFAR	3	0	-.2228124	.3648765E-01	-6.106516	.9102812E-06
4	MA_SEAS	4	12	-.5542739	.1936661	-2.862008	.7478265E-02

SOUTHERN CALIFORNIA RAPID TRAN DIST, LOS ANGELES, CA -- OFFPEAK PERIOD Mon 09-11-1989

DEPENDENT VARIABLE 14 LUTRIPS
FROM 1983: 8 UNTIL 1987: 6
TOTAL OBSERVATIONS 47 SKIPPED/MISSING 0
USABLE OBSERVATIONS 47 DEGREES OF FREEDOM 43
R**2 .81241896 RBAR**2 .79933192
SSR .35934433E-01 SEE .28908208E-01
DURBIN-WATSON 2.08186301
Q(18)= 10.1170 SIGNIFICANCE LEVEL .928020

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	0	0	4.975022	1.205591	4.126623	.1657073E-03
2	LAVFARE	18	0	-.2389947	.5124445E-01	-4.663815	.3019694E-04
3	LVMILES	16	0	.7376344	.7985360E-01	9.237334	.3728049E-08

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
4	RHO	1	0	.8650368	.6811759E-01	12.69917	.3718988E-08

MADISON METRO, MADISON, WI -- SYSTEM TOTAL Wed 10-04-1989

CONVERGENCE REACHED ON ITERATION 12
DEPENDENT VARIABLE 33 FUTRIPS
FROM 1986: 9 UNTIL 1988: 4
TOTAL OBSERVATIONS 20 SKIPPED/MISSING 0
USABLE OBSERVATIONS 20 DEGREES OF FREEDOM 13
R**2 .88666118 RBAR**2 .83435095
SSR .83288667E-02 SEE .25311698E-01
DURBIN-WATSON 1.89187678
Q(10)= 9.17996 SIGNIFICANCE LEVEL .515115

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	-.3766891E-02	.4012458E-02	-.9387989	.3649502
2	N_FVMILE	2	0	.9385385	.1387656	6.763479	.1335002E-04
3	N_FAVFAR	3	1	-.1792830	.7285601E-01	-2.460786	.2862664E-01
4	N_FAVFAR	4	3	-.2214139	.9478434E-01	-2.335976	.3615848E-01
5	N_FEMPLO	5	2	2.679763	1.001074	2.676889	.1901322E-01
6	N_FGAS	6	6	.1608918	.5766277E-01	2.790219	.1531371E-01
7	AR	7	1	-.6042468	.2437048	-2.479422	.2764012E-01

NASHVILLE TRANSIT AUTHORITY, NASHVILLE, TN -- SYSTEM TOTAL Tue 08-15-1989

CONVERGENCE REACHED ON ITERATION 9
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1984: 6 UNTIL 1986: 9
TOTAL OBSERVATIONS 28 SKIPPED/MISSING 0
USABLE OBSERVATIONS 28 DEGREES OF FREEDOM 21
R**2 .86060315 RBAR**2 .82077548
SSR .26733427E-01 SEE .35679411E-01
DURBIN-WATSON 1.96578184
Q(14)= 14.4658 SIGNIFICANCE LEVEL .415616

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	-.5236523E-02	.2733912E-02	-1.915396	.6915958E-01
2	N_FVMILE	2	0	.6926982	.1663754	4.163466	.4395489E-03
3	N_FAVFAR	3	1	-.2413439	.7429501E-01	-3.248453	.3845217E-02
4	N_FAVFAR	4	2	-.2852204	.7383366E-01	-3.863013	.9010143E-03
5	N_FGAS	5	5	.2963903	.9118093E-01	3.250573	.3826290E-02
6	AR	6	1	-1.142275	.2118931	-5.390805	.2396733E-04
7	AR	7	2	-.3493722	.2116393	-1.650791	.1136585

N. SAN DIEGO CO. TRAN. DIST., OCEANSIDE, CA -- SYSTEM TOTAL Fri 08-11-1989

DEPENDENT VARIABLE 15 LUTRIPS
FROM 1984: 8 UNTIL 1987: 6
TOTAL OBSERVATIONS 35 SKIPPED/MISSING 0
USABLE OBSERVATIONS 35 DEGREES OF FREEDOM 29
R**2 .72970273 RBAR**2 .68309975
SSR .22480253E-01 SEE .27842075E-01
DURBIN-WATSON 1.76067680
Q(15)= 11.5198 SIGNIFICANCE LEVEL .714953

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL	
1	CONSTANT	0	0	.5496049	1.799100	.3054888	.7621760	
2	LVMILES	17	0	.8575048	.1271984	6.741474	.2168115E-06	
3	LAVFARE	19	0	-.3503479	.1327729	-2.638701	.1324779E-01	
4	LGAS	21	0	.2198458	.8640882E-01	2.544252	.1653742E-01	
5	MONTH	10	0	-.3519307E-02	.1601486E-02	-2.197525	.3612053E-01	Jan=1,Feb=2,...,Dec=12

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
6	RHO	1	0	.5854476	.1565884	3.738768	.8091649E-03

OSHKOSH TRANSIT SYSTEM, OSHKOSH, WI -- SYSTEM TOTAL Wed 08-23-1989

CONVERGENCE REACHED ON ITERATION 19
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1985: 8 UNTIL 1987:12
TOTAL OBSERVATIONS 29 SKIPPED/MISSING 0
USABLE OBSERVATIONS 29 DEGREES OF FREEDOM 24
R**2 .87630114 RBAR**2 .85568466
SSR .60997057E-01 SEE .50413729E-01
DURBIN-WATSON 1.86196989
Q(14)= 16.6207 SIGNIFICANCE LEVEL .276954

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	N_FVMILE	1	0	1.973688	.1759944	11.21449	.5008617E-10
2	N_FAVFAR	2	1	-.1670763	.5410897E-01	-3.087774	.5032938E-02
3	N_FGAS	3	4	.3586584	.1309175	2.739575	.1141911E-01
4	N_FEMPLO	4	5	2.812965	1.725270	1.630449	.1160621
5	AR	5	1	.2327362	.2088011	1.114631	.2760464

PHOENIX TRANSIT SYTEM, PHOENIX, AZ -- SYSTEM TOTAL Fri 08-11-1989

CONVERGENCE REACHED ON ITERATION 12
DEPENDENT VARIABLE 30 FUTRIPS
FROM 1983:12 UNTIL 1986: 6
TOTAL OBSERVATIONS 31 SKIPPED/MISSING 0
USABLE OBSERVATIONS 31 DEGREES OF FREEDOM 25
R**2 .71345289 RBAR**2 .65614347
SSR .12894170 SEE .71816905E-01
DURBIN-WATSON 1.86463570
Q(15)= 14.4995 SIGNIFICANCE LEVEL .488036

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	-.3987791E-02	.6053324E-02	-.6587770	.5160611
2	N_FVMILE	2	0	1.167478	.2748464	4.247745	.2616654E-03
3	N_FAVFAR	3	0	-.3214516	.1732495	-1.855426	.7536363E-01
4	N_FGAS	4	2	.6537297	.2799636	2.335052	.2786725E-01
5	AR	5	1	-.8292858	.1952131	-4.248105	.2614223E-03
6	AR	6	2	-.3311268	.1978609	-1.673533	.1066909

TRI-COUNTY METRO TRANSP DIST OF OREGON, PORTLAND, OR -- SYSTEM TOTAL Fri 09-15-1989

CONVERGENCE REACHED ON ITERATION 14
DEPENDENT VARIABLE 36 FUTRIPS
FROM 1985: 3 UNTIL 1987: 8
TOTAL OBSERVATIONS 30 SKIPPED/MISSING 0
USABLE OBSERVATIONS 30 DEGREES OF FREEDOM 23
R**2 .71467524 RBAR**2 .64024269
SSR .28560222E-01 SEE .35238456E-01
DURBIN-WATSON 1.98926529
Q(15)= 9.54905 SIGNIFICANCE LEVEL .847107

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL	
***	*******	***	***	************	************	************	************	
1	CONSTANT	1	0	.6350228E-03	.4552501E-02	.1394888	.8902782	
2	N_FVMILE	2	0	.4835178	.2061273	2.345724	.2798102E-01	
3	N_FAVFAR	3	0	-.3867207	.8985849E-01	-4.303663	.2641550E-03	
4	N_FEMPLO	4	2	.9517976	.6412509	1.484283	.1513105	
5	N_FGAS	5	4	.3279172	.1303966	2.514769	.1935259E-01	
6	N_FRAIL	6	0	-.3911133	.1672190	-2.338929	.2839261E-01	light rail 9/86-8/87=1
7	AR	7	1	-.5042564	.1912417	-2.636750	.1474239E-01	

TRI-COUNTY METRO TRANSP DIST OF OREGON, PORTLAND, OR -- PEAK PERIOD Fri 09-15-1989

CONVERGENCE REACHED ON ITERATION 2
DEPENDENT VARIABLE 36 FUTRIPS
FROM 1985: 3 UNTIL 1987: 8
TOTAL OBSERVATIONS 30 SKIPPED/MISSING 0
USABLE OBSERVATIONS 30 DEGREES OF FREEDOM 25
R**2 .39790430 RBAR**2 .30156899
SSR .67701562E-01 SEE .52039048E-01
DURBIN-WATSON 2.37972089
Q(15)= 16.6927 SIGNIFICANCE LEVEL .337567

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL	
1	N_FVMILE	1	0	.4796434	.2699918	1.776511	.8782143E-01	
2	N_FAVFAR	2	0	-.1912452	.1089960	-1.754608	.9157726E-01	
3	N_FRAIL	3	0	-.5231963	.2116377	-2.472132	.2058906E-01	light rail 9/86-8/87=1
4	N_FEMPLO	4	2	1.279597	.8929892	1.432937	.1642635	
5	N_FGAS	5	5	.3569661	.2269120	1.573148	.1282563	

TRI-COUNTY METRO TRANSP DIST OF OREGON, PORTLAND, OR -- OFF PEAK Fri 09-15-1989

CONVERGENCE REACHED ON ITERATION 15
DEPENDENT VARIABLE 36 FUTRIPS
FROM 1985: 2 UNTIL 1987: 8
TOTAL OBSERVATIONS 31 SKIPPED/MISSING 0
USABLE OBSERVATIONS 31 DEGREES OF FREEDOM 23
R**2 .66788852 RBAR**2 .56681112
SSR .42570137E-01 SEE .43021803E-01
DURBIN-WATSON 2.27751802
Q(15)= 16.1181 SIGNIFICANCE LEVEL .374246

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL	
1	CONSTANT	1	0	.1435330E-02	.3238352E-02	.4432285	.6617416	
2	N_FVMILE	2	0	.7155144	.1330741	5.376811	.1841146E-04	
3	N_FAVFAR	3	0	-.4292495	.1164127	-3.687309	.1218760E-02	
4	N_FRAIL	4	0	-.3980526	.1894896	-2.100657	.4683435E-01	light rail 9/86-8/87=1
5	N_FGAS	5	4	.5279080	.1614506	3.269781	.3366144E-02	
6	N_FEMPLO	6	2	.6822414	.3094016	2.205035	.3772112E-01	
7	MA_SEAS	7	12	-1.008915	.2400639	-4.202692	.3397522E-03	
8	MA_SEAS	8	24	-1.467476	.6163910	-2.380755	.2594468E-01	

GREATER RICHMOND TRANSIT CO., RICHMOND, VA -- SYSTEM TOTAL Tue 08-15-1989

CONVERGENCE REACHED ON ITERATION 2
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1985: 2 UNTIL 1987:10
TOTAL OBSERVATIONS 33 SKIPPED/MISSING 0
USABLE OBSERVATIONS 33 DEGREES OF FREEDOM 28
R**2 .73945788 RBAR**2 .70223758
SSR .27167596E-01 SEE .31149178E-01
DURBIN-WATSON 2.58123244
Q(15)= 8.96457 SIGNIFICANCE LEVEL .879363

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	-.2659812E-02	.5522973E-02	-.4815906	.6338395
2	N_FVMILE	2	0	1.294612	.1732256	7.473561	.3860064E-07
3	N_FAVFAR	3	0	-.6235998	.2561804	-2.434222	.2155541E-01
4	N_FGAS	4	2	.1636779	.8358975E-01	1.958110	.6024877E-01
5	N_FEMPLO	5	1	1.672895	.8234563	2.031553	.5178733E-01

RIVERSIDE TRANSIT AGENCY, RIVERSIDE, CA -- SYSTEM TOTAL Tue 08-15-1989

CONVERGENCE REACHED ON ITERATION 7
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1986: 3 UNTIL 1988: 4
TOTAL OBSERVATIONS 26 SKIPPED/MISSING 0
USABLE OBSERVATIONS 26 DEGREES OF FREEDOM 21
R**2 .80235702 RBAR**2 .76471074
SSR .97598373E-02 SEE .21558158E-01
DURBIN-WATSON 2.05097783
Q(13)= 8.88017 SIGNIFICANCE LEVEL .781932

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	.4986694E-02	.2378220E-02	2.096817	.4829765E-01
2	N_FVMILE	2	0	.8196769	.1049025	7.813702	.1236208E-06
3	N_FAVFAR	3	0	-.1194350	.3012278E-01	-3.964940	.7064741E-03
4	N_FEMPLO	4	0	.9075490	.4875361	1.861501	.7673001E-01
5	AR	5	1	-.8477486	.1288137	-6.581197	.1621712E-05

SACRAMENTO REGIONAL TRANSIT DISTRICT, SACRAMENTO, CA -- SYSTEM TOTAL Tue 09-12-1989

CONVERGENCE REACHED ON ITERATION 21
DEPENDENT VARIABLE 36 FUTRIPS
FROM 1985: 4 UNTIL 1988: 1
TOTAL OBSERVATIONS 34 SKIPPED/MISSING 0
USABLE OBSERVATIONS 34 DEGREES OF FREEDOM 27
R**2 .86837010 RBAR**2 .83911901
SSR .31376090E-01 SEE .34089257E-01
DURBIN-WATSON 2.04283773
Q(15)= 17.2122 SIGNIFICANCE LEVEL .306337

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL	
1	CONSTANT	1	0	.5833775E-04	.9964058E-03	.5854818E-01	.9537432	
2	N_FVMILE	2	0	1.050355	.6084927E-01	17.26159	.3718988E-08	
3	N_FAVFAR	3	0	-.1615935	.2131542E-01	-7.581060	.4102953E-07	
4	N_FGAS	4	0	.2181660	.5484441E-01	3.977908	.4691949E-03	
5	N_FRAIL	5	0	-.1710480E-01	.3251529E-02	-5.260539	.1513683E-04	light rail 3/87-1/88=1
6	AR	6	1	-.7834186	.1362808	-5.748564	.4117814E-05	
7	MA_SEAS	7	12	-1.697819	.2533853	-6.700542	.3454379E-06	

SACRAMENTO REGIONAL TRANSIT DISTRICT, SACRAMENTO, CA -- PEAK PERIOD Tue 09-12-1989

CONVERGENCE REACHED ON ITERATION 12
DEPENDENT VARIABLE 36 FUTRIPS
FROM 1985: 4 UNTIL 1988: 1
TOTAL OBSERVATIONS 34 SKIPPED/MISSING 0
USABLE OBSERVATIONS 34 DEGREES OF FREEDOM 27
R**2 .86481068 RBAR**2 .83476860
SSR .39936326E-01 SEE .38459370E-01
DURBIN-WATSON 1.87597554
Q(15)= 8.85472 SIGNIFICANCE LEVEL .884990

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL	
1	CONSTANT	1	0	.9706962E-03	.1771554E-02	.5479349	.5882384	
2	N_FVMILE	2	0	.7064316	.5571285E-01	12.67987	.3719684E-08	
3	N_FAVFAR	3	0	-.2231231	.3819841E-01	-5.841163	.3222277E-05	
4	N_FGAS	4	0	.1374687	.9643706E-01	1.425475	.1654789	
5	N_FRAIL	5	0	-.1492423E-01	.3129134E-02	-4.769445	.5660248E-04	light rail 3/87-1/88=1
6	AR	6	1	-.6371876	.1499791	-4.248509	.2286732E-03	
7	MA_SEAS	7	12	-1.215484	.1789578	-6.792015	.2740558E-06	

SACRAMENTO REGIONAL TRANSIT DISTRICT, SACRAMENTO, CA -- OFFPEAK PERIOD

CONVERGENCE REACHED ON ITERATION 14
DEPENDENT VARIABLE 36 FUTRIPS
FROM 1985: 4 UNTIL 1988: 1
TOTAL OBSERVATIONS 34 SKIPPED/MISSING 0
USABLE OBSERVATIONS 34 DEGREES OF FREEDOM 27
R**2 .72067664 RBAR**2 .65860478
SSR .63883827E-01 SEE .48642242E-01
DURBIN-WATSON 1.86286804
Q(15)= 15.9196 SIGNIFICANCE LEVEL .387415

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL	
***	*******	***	***	************	************	************	************	
1	CONSTANT	1	0	-.1832295E-02	.2144175E-02	-.8545452	.4003220	
2	N_FVMILE	2	0	1.369814	.1619143	8.460115	.8226102E-08	
3	N_FAVFAR	3	0	-.1295343	.4705734E-01	-2.752690	.1043688E-01	
4	N_FGAS	4	0	.3458850	.1173580	2.947263	.6536037E-02	
5	N_FRAIL	5	0	-.1364141E-01	.4533971E-02	-3.008712	.5624245E-02	light rail 3/87-1/88=1
6	AR	6	1	-.5471836	.1707933	-3.203776	.3466596E-02	
7	MA_SEAS	7	12	-1.319061	.2032595	-6.489544	.5924388E-06	

SAN DIEGO TRANSIT CORP., SAN DIEGO, CA -- SYSTEM TOTAL Mon 08-14-1989

CONVERGENCE REACHED ON ITERATION 39
DEPENDENT VARIABLE 29 FUTRIPS
FROM 1986: 2 UNTIL 1988: 4
TOTAL OBSERVATIONS 27 SKIPPED/MISSING 0
USABLE OBSERVATIONS 27 DEGREES OF FREEDOM 22
R**2 .71016094 RBAR**2 .65746293
SSR .12562925E-01 SEE .23896486E-01
DURBIN-WATSON 1.71993002
Q(13)= 9.60684 SIGNIFICANCE LEVEL .725725

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	N_FVMILE	1	0	.4930783	.1344882	3.666331	.1356087E-02
2	N_FAVFAR	2	0	-.2703127	.1461741	-1.849252	.7790381E-01
3	N_FEMPLO	3	0	4.851134	1.856817	2.612608	.1589498E-01
4	MA	4	1	-.7296425	.1563082	-4.667973	.1180616E-03
5	MA_SEAS	5	12	-1.237189	.3055326	-4.049286	.5348458E-03

GOLDEN GATE BRIDGE, HIWAY & TRANSP DIST., SAN FRANCISCO, CA -- SYSTEM TOTAL Tue 08-29-1989

CONVERGENCE REACHED ON ITERATION 2
DEPENDENT VARIABLE 33 FUTRIPS
FROM 1983: 5 UNTIL 1985: 6
TOTAL OBSERVATIONS 26 SKIPPED/MISSING 0
USABLE OBSERVATIONS 26 DEGREES OF FREEDOM 22
R**2 .89168243 RBAR**2 .87691186
SSR .19796434E-01 SEE .29997299E-01
DURBIN-WATSON 1.86040870
Q(13)= 6.53918 SIGNIFICANCE LEVEL .924315

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	-.2762920E-02	.5987563E-02	-.4614431	.6490107
2	N_FVMILE	2	0	1.128700	.1175236	9.604022	.2504890E-08
3	N_FAVFAR	3	0	-.1510069	.6618616E-01	-2.281548	.3255216E-01
4	N_FEMPLO	4	1	2.151458	.6948190	3.096430	.5268900E-02

GOLDEN GATE BRIDGE, HIWAY & TRANSP DIST., SAN FRANCISCO, CA -- PEAK PERIOD Tue 08-29-1989

CONVERGENCE REACHED ON ITERATION 13
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1983: 8 UNTIL 1985: 6
TOTAL OBSERVATIONS 23 SKIPPED/MISSING 0
USABLE OBSERVATIONS 23 DEGREES OF FREEDOM 19
R**2 .96489656 RBAR**2 .95935392
SSR .63473905E-02 SEE .18277669E-01
DURBIN-WATSON 1.60362564
Q(11)= 9.94104 SIGNIFICANCE LEVEL .535698

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	.1324675E-02	.2049305E-02	.6464023	.5257507
2	N_FVMILE	2	0	.8805526	.5613451E-01	15.68647	.3721487E-08
3	N_FAVFAR	3	0	-.1394384	.4518116E-01	-3.086206	.6079108E-02
4	MA	4	1	-.4969038	.2385966	-2.082610	.5103952E-01

GOLDEN GATE BRIDGE, HIWAY & TRANSP DIST., SAN FRANCISCO, CA - OFF PEAK Tue 08-29-1989

CONVERGENCE REACHED ON ITERATION 2
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1983: 9 UNTIL 1985: 6
TOTAL OBSERVATIONS 22 SKIPPED/MISSING 0
USABLE OBSERVATIONS 22 DEGREES OF FREEDOM 19
R**2 .82159777 RBAR**2 .80281859
SSR .66649535E-01 SEE .59227276E-01
DURBIN-WATSON 2.00379296
Q(11)= 5.43848 SIGNIFICANCE LEVEL .908089

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	N_FVMILE	1	0	1.642253	.5138405	3.196037	.4756256E-02
2	N_FAVFAR	2	0	-.3115261	.1198453	-2.599401	.1760771E-01
3	N_FEMPLO	3	1	4.252459	1.528591	2.781947	.1188148E-01

SANTA CLARA COUNTY TRANSIT, SAN JOSE, CA -- SYSTEM TOTAL Tue 10-03-1989

CONVERGENCE REACHED ON ITERATION 2
DEPENDENT VARIABLE 29 FUTRIPS
FROM 1986: 5 UNTIL 1988: 4
TOTAL OBSERVATIONS 24 SKIPPED/MISSING 0
USABLE OBSERVATIONS 24 DEGREES OF FREEDOM 19
R**2 .60296470 RBAR**2 .51937832
SSR .36717863E-01 SEE .43960426E-01
DURBIN-WATSON 2.20401175
Q(12)= 7.12384 SIGNIFICANCE LEVEL .849322

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	-.6352830E-02	.9899031E-02	-.6417628	.5286944
2	N_FVMILE	2	0	.8214694	.2809672	2.923720	.8712120E-02
3	N_FAVFAR	3	0	-.4603714	.2122005	-2.169511	.4293429E-01
4	N_FEMPLO	4	1	1.966586	1.282905	1.532916	.1417791
5	N_FGAS	5	3	.3411523	.1585728	2.151393	.4452054E-01

SARASOTA COUNTY AREA TRANSIT, SARASOTA, FL -- SYSTEM TOTAL Tue 08-15-1989

CONVERGENCE REACHED ON ITERATION 13
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1986: 1 UNTIL 1988: 5
TOTAL OBSERVATIONS 29 SKIPPED/MISSING 0
USABLE OBSERVATIONS 29 DEGREES OF FREEDOM 24
R**2 .72765421 RBAR**2 .68226325
SSR .29544873E-01 SEE .35086128E-01
DURBIN-WATSON 2.12542344
Q(14)= 5.57924 SIGNIFICANCE LEVEL .976009

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	-.1136985E-02	.4959296E-02	-.2292634	.8206068
2	N_FVMILE	2	0	.9204644	.1685941	5.459647	.1304259E-04
3	N_FAVFAR	3	0	-.2142694	.8016757E-01	-2.672769	.1331085E-01
4	N_FEMPLO	4	2	1.570097	.7381566	2.127051	.4389215E-01
5	MA_SEAS	5	12	-.4641672	.2372085	-1.956789	.6210339E-01

SEATTLE METRO, SEATTLE, WA -- SYSTEM TOTAL Mon 08-14-1989

CONVERGENCE REACHED ON ITERATION 25
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1984: 4 UNTIL 1987: 1
TOTAL OBSERVATIONS 34 SKIPPED/MISSING 0
USABLE OBSERVATIONS 34 DEGREES OF FREEDOM 29
R**2 .87770517 RBAR**2 .86083691
SSR .11072093E-01 SEE .19539608E-01
DURBIN-WATSON 1.81384053
Q(15)= 17.0125 SIGNIFICANCE LEVEL .318117

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	-.1331443E-02	.1490777E-02	-.8931203	.3791447
2	N_FVHOUR	2	0	1.013062	.9877407E-01	10.25635	.3756037E-08
3	N_FAVFAR	3	0	-.2658345	.1130144	-2.352217	.2566231E-01
4	AR	4	1	-.4796762	.1739668	-2.757286	.9978314E-02
5	MA_SEAS	5	12	-.6236649	.2105527	-2.962036	.6044207E-02

SOUTH BEND TRANSPO, SOUTH BEND, IN -- SYSTEM TOTAL Tue 08-15-189

CONVERGENCE REACHED ON ITERATION 9
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1982: 3 UNTIL 1984:12
TOTAL OBSERVATIONS 34 SKIPPED/MISSING 0
USABLE OBSERVATIONS 34 DEGREES OF FREEDOM 29
R**2 .70156537 RBAR**2 .66040197
SSR .42308105E-01 SEE .38195552E-01
DURBIN-WATSON 1.94689110
Q(15)= 20.5794 SIGNIFICANCE LEVEL .150813

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	N_FVMILE	1	0	.7933388	.1696839	4.675393	.6248503E-04
2	N_FAVFAR	2	0	-.2606330	.5688171E-01	-4.582018	.8093100E-04
3	N_FEMPLO	3	0	.5319538	.3602213	1.476742	.1505227
4	N_FGAS	4	1	.5248215	.1353244	3.878248	.5562495E-03
5	MA	5	1	-.7633170	.1777441	-4.294471	.1789143E-03

EAST VOLUSIA TRANSP. AUTH, SOUTH DAYTONA, FL - SYSTEM TOTAL Wed 10-04-1989

CONVERGENCE REACHED ON ITERATION 11
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1986: 2 UNTIL 1988: 4
TOTAL OBSERVATIONS 27 SKIPPED/MISSING 0
USABLE OBSERVATIONS 27 DEGREES OF FREEDOM 21
R**2 .68571332 RBAR**2 .61088316
SSR .20091658E-01 SEE .30931305E-01
DURBIN-WATSON 1.95555975
Q(13)= 14.3503 SIGNIFICANCE LEVEL .349643

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	.3192353E-02	.4538991E-02	.7033177	.4895827
2	N_FVMILE	2	0	.7037336	.1714132	4.105480	.5049286E-03
3	N_FAVFAR	3	0	-.4226760	.1469284	-2.876748	.9027287E-02
4	N_FEMPLO	4	2	1.773135	.9830380	1.803730	.8563984E-01
5	N_FGAS	5	0	.1492186	.8085889E-01	1.845420	.7912499E-01
6	AR	6	1	-.3367869	.2127807	-1.582789	.1284145

SPOKANE TRANSIT AUTHORITY, SPOKANE, WA -- SYSTEM TOTAL Fri 09-15-1989

CONVERGENCE REACHED ON ITERATION 18
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1983: 5 UNTIL 1985:12
TOTAL OBSERVATIONS 32 SKIPPED/MISSING 0
USABLE OBSERVATIONS 32 DEGREES OF FREEDOM 23
R**2 .77088812 RBAR**2 .69119703
SSR .22883318E-01 SEE .31542461E-01
DURBIN-WATSON 2.10735091
Q(15)= 22.1232 SIGNIFICANCE LEVEL .104614

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	.9880745E-03	.1179596E-02	.8376378	.4108543
2	N_FVMILE	2	0	.1331804	.3250009E-01	4.097848	.4410920E-03
3	N_FVMILE	3	1	.1085543	.3106237E-01	3.494722	.1953026E-02
4	N_FAVFAR	4	0	-.2953405	.5194791E-01	-5.685322	.8670886E-05
5	D_FAVFAR	5	1	.4391807	.1395085	3.148057	.4503256E-02
6	N_FGAS	6	1	.2165135	.9547529E-01	2.267744	.3305087E-01
7	AR	7	1	-1.052031	.1336678	-7.870493	.6040600E-07
8	AR	8	2	-.7465142	.1392394	-5.361372	.1912250E-04
9	MA_SEAS	9	12	-1.140421	.1963927	-5.806836	.6461127E-05

SPOKANE TRANSIT AUTHORITY, SPOKANE, WA -- PEAK PERIOD Fri 09-15-1989

CONVERGENCE REACHED ON ITERATION 10
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1983: 5 UNTIL 1985:12
TOTAL OBSERVATIONS 32 SKIPPED/MISSING 0
USABLE OBSERVATIONS 32 DEGREES OF FREEDOM 28
R**2 .62750576 RBAR**2 .58759566
SSR .38600082E-01 SEE .37129158E-01
DURBIN-WATSON 1.95290606
Q(15)= 9.46188 SIGNIFICANCE LEVEL .852156

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	N_FVMILE	1	0	.1880763	.7524660E-01	2.499466	.1857368E-01
2	N_FAVFAR	2	3	-.3218734	.7691818E-01	-4.184620	.2556620E-03
3	N_FGAS	3	1	.3793499	.1277919	2.968498	.6071789E-02
4	MA	4	1	-.7776011	.1689057	-4.603757	.8188961E-04

SPOKANE TRANSIT AUTH, SPOKANE, WA -- OFF-PEAK PERIOD Fri 09-15-1989

CONVERGENCE REACHED ON ITERATION 15
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1983: 3 UNTIL 1985:12
TOTAL OBSERVATIONS 34 SKIPPED/MISSING 0
USABLE OBSERVATIONS 34 DEGREES OF FREEDOM 27
R**2 .70922640 RBAR**2 .64461004
SSR .43240845E-01 SEE .40018905E-01
DURBIN-WATSON 2.00228849
Q(15)= 19.3351 SIGNIFICANCE LEVEL .198949

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	-.2934986E-02	.3740533E-02	-.7846438	.4394930
2	N_FVMILE	2	0	.1902672	.4679364E-01	4.066090	.3715009E-03
3	N_FVMILE	3	1	.1337727	.4833009E-01	2.767896	.1006654E-01
4	N_FAVFAR	4	0	-.2785735	.8926468E-01	-3.120759	.4264658E-02
5	N_FAVFAR	5	1	-.4633721	.8640771E-01	-5.362624	.1151626E-04
6	N_FGAS	6	1	.4200283	.1895739	2.215644	.3533583E-01
7	MA_SEAS	7	12	-.9616361	.1882686	-5.107788	.2280289E-04

CITY UTILITIES TRANSP. DEPT, SPRINGFIELD, MO -- SYSTEM TOTAL Tue 10-03-1989

CONVERGENCE REACHED ON ITERATION 22
DEPENDENT VARIABLE 32 FUTRIPS
FROM 1982: 5 UNTIL 1984:12
TOTAL OBSERVATIONS 32 SKIPPED/MISSING 0
USABLE OBSERVATIONS 32 DEGREES OF FREEDOM 25
R**2 .71540778 RBAR**2 .64710564
SSR .28086772E-01 SEE .33518217E-01
DURBIN-WATSON 2.06437240
Q(15)= 6.79386 SIGNIFICANCE LEVEL .963119

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	1	0	-.1349834E-02	.4226230E-02	-.3193945	.7520811
2	N_FVMILE	2	0	.2255022	.1455454	1.549359	.1338631
3	N_FAVFAR	3	0	-.4806515	.5605466E-01	-8.574693	.1020181E-07
4	N_FGAS	4	1	.1647273	.9542138E-01	1.726315	.9663071E-01
5	AR	5	1	-.3522406	.1746053	-2.017354	.5451584E-01
6	AR	6	2	-.5510612	.1750632	-3.147785	.4221950E-02
7	MA_SEAS	7	12	.5579408	.2410237	2.314879	.2911822E-01

CENTRE AREA TRANSP. AUTH, STATE COLLEGE, PA -- SYSTEM TOTAL Wed 10-04-1989

DEPENDENT VARIABLE 14 LUTRIPS
FROM 1984:11 UNTIL 1987: 2
TOTAL OBSERVATIONS 28 SKIPPED/MISSING 0
USABLE OBSERVATIONS 28 DEGREES OF FREEDOM 23
R**2 .90641419 RBAR**2 .89013840
SSR .35285295 SEE .12386054
DURBIN-WATSON 1.72598374
Q(14)= 13.7189 SIGNIFICANCE LEVEL .470854

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	0	0	-17.53385	5.533399	-3.168730	.4286931E-02
2	LVMILES	16	0	1.618710	.3704849	4.369168	.2243419E-03
3	LAVFARE	18	0	-.6422149	.1404341	-4.573069	.1349076E-03
4	LGAS	20	4	.7821688	.1951711	4.007606	.5520189E-03
5	LEMPLO	21	0	1.963606	.8589022	2.286181	.3178163E-01

PIERCE TRANSIT, TACOMA, WA -- SYSTEM TOTAL Tue 08-15-1989

CONVERGENCE REACHED ON ITERATION 27
DEPENDENT VARIABLE 33 FUTRIPS
FROM 1983: 5 UNTIL 1985:12
TOTAL OBSERVATIONS 32 SKIPPED/MISSING 0
USABLE OBSERVATIONS 32 DEGREES OF FREEDOM 26
R**2 .68579456 RBAR**2 .62537043
SSR .72002022E-01 SEE .52624220E-01
DURBIN-WATSON 1.79205471
Q(15)= 9.53465 SIGNIFICANCE LEVEL .847947

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	.2906843E-02	.4614020E-02	.6300022	.5341912
2	N_FVMILE	2	0	.8496965	.3114184	2.728472	.1125514E-01
3	N_FAVFAR	3	0	-.4323256	.9191079E-01	-4.703753	.7346811E-04
4	N_FGAS	4	1	.4532048	.1725582	2.626388	.1427404E-01
5	AR	5	1	-.4966286	.1987288	-2.499027	.1909948E-01
6	AR	6	2	-.5742301	.1991314	-2.883674	.7791052E-02

TOLEDO AREA REGIONAL TRANSIT AUTH., TOLEDO, OH -SYSTEM TOTAL Tue 10-03-1989

DEPENDENT VARIABLE 14 LUTRIPS
FROM 1985: 3 UNTIL 1988: 4
TOTAL OBSERVATIONS 38 SKIPPED/MISSING 0
USABLE OBSERVATIONS 38 DEGREES OF FREEDOM 34
R**2 .97674654 RBAR**2 .97469477
SSR .90889620E-01 SEE .51703231E-01
DURBIN-WATSON 2.10571080
Q(18)= 8.42052 SIGNIFICANCE LEVEL .971697

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	0	0	.1838450	.7979221	.2304048	.8191580
2	LVMILES	16	0	.8708597	.6237547E-01	13.96157	.1153019E-14
3	LAVFARE	18	0	-.8547404	.2893641E-01	-29.53858	.6822153E-16

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
4	RHO	1	0	.2627290	.1685595	1.558672	.1283348

WILLIAMSPORT BUREAU OF TRANS, WILLIAMSPORT, PA -SYSTEM TOTAL Wed 08-23-1989

CONVERGENCE REACHED ON ITERATION 13
DEPENDENT VARIABLE 33 FUTRIPS
FROM 1985:10 UNTIL 1988: 5
TOTAL OBSERVATIONS 32 SKIPPED/MISSING 0
USABLE OBSERVATIONS 32 DEGREES OF FREEDOM 27
R**2 .78420960 RBAR**2 .75224066
SSR .99818739E-02 SEE .19227559E-01
DURBIN-WATSON 2.32174664
Q(15)= 10.6476 SIGNIFICANCE LEVEL .777149

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
1	CONSTANT	1	0	-.1493218E-03	.2237549E-02	-.6673455E-01	.9472847
2	N_FVMILE	2	0	.8264578	.8115224E-01	10.18404	.3814860E-08
3	N_FAVFAR	3	0	-.2993627	.1187812	-2.520286	.1794335E-01
4	N_FEMPLO	4	3	.8387924	.3245033	2.584850	.1546680E-01
5	MA_SEAS	5	12	-.6759709	.2540004	-2.661299	.1294607E-01

PALM BEACH COUNTY TRANS AUTH, WEST PALM BEACH, FL -SYSTEM TOTAL Tue 08-15-1989

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DEPENDENT VARIABLE   14      LUTRIPS
FROM    1985: 4 UNTIL     1988: 1
TOTAL OBSERVATIONS      34      SKIPPED/MISSING          0
USABLE OBSERVATIONS     34      DEGREES OF FREEDOM      29
R**2                .87657669   RBAR**2          .85955278
SSR            .37708537E-01    SEE          .36059595E-01
DURBIN-WATSON  2.37699841
Q(  15)=    14.0863         SIGNIFICANCE LEVEL  .518994
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NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
1	CONSTANT	0	0	-5.986112	3.059698	-1.956439	.6010512E-01
2	LVMILES	16	0	.9202855	.1279354	7.193359	.6794576E-07
3	LEMPLO	21	0	1.069849	.4228498	2.530091	.1709124E-01
4	LAVFARE	18	0	-.6046581	.2069095	-2.922331	.6668761E-02

NO.	LABEL	VAR	LAG	COEFFICIENT	STAND. ERROR	T-STATISTIC	SIGNIF LEVEL
***	*******	***	***	************	************	************	************
5	RHO	1	0	.9107384	.7385363E-01	12.33167	.3719454E-08

END OF LISTING

APPENDIX D

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