17. APTA PR-M-S-017-06 Standard for Definition and Measurement of Wheel Tread Taper

Approved May 18, 2007
APTA PRESS Task Force

Authorized June 2, 2007
APTA Commuter Rail Executive Committee

Abstract: This standard provides a definition and practice for the measurement of wheel tread taper on wheels used in railroad passenger service in relation to vehicle and truck stability, and guidance to the maintenance of the wheel profile acceptable for safe dynamic performance.

Key Words: wheel, taper, conicity, stability, hunting

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Standard for Definition and Measurement of Wheel Tread Taper (Conicity)

1. Overview

1.1 Scope

This standard applies to railroad passenger equipment of all types, including non-passenger-carrying cars and locomotives that are intended for use in passenger service on the general railroad system in the United States. This standard does not apply to trucks or vehicles that are equipped with independently rotating wheels.

1.2 Purpose

The purpose of this standard is to provide a standard definition and practice for the measurement of wheel tread taper or conicity which is a major factor governing vehicle and truck stability and curving. The standard also provides guidance to the selection and maintenance of wheel and rail profiles acceptable for safe dynamic performance.

2. References

This standard, where applicable, shall be used in conjunction with the following publications. If the following publications are superseded by an approved revision, the approved revision shall apply.

APTA PR-M-RP-009, Recommended Practice for New Truck Design Process

3. Definitions, abbreviations and acronyms

3.1 Definitions

3.1.1 critical speed: The vehicle speed above which hunting typically occurs for a given truck.

3.1.2 contact angle: The angle of a tangent line at the point of contact between the wheel and rail with respect to the axis of the wheelset.

3.1.3 flange clearance: The maximum lateral distance a wheelset can shift from its centered position between the rails to a point at which the angle of contact between the wheel and rail does not exceed 25 degrees with respect to the wheelset axis.

3.1.4 rail rollover: The occurrence of a rail rolling about its base corner as a result of a net overturning moment applied to the rail by the combination of lateral and vertical forces acting between the wheel and rail.

3.1.5 rolling radius: The rolling radius of a wheel, measured as the perpendicular (radial) distance between the wheel/axle center of rotation and the point of contact with the rail. Rolling radius may vary with respect to the lateral location of the point of rolling contact.

3.1.6 rolling radius difference: The difference between the rolling radius of the left wheel and the rolling radius of the right wheel of a wheelset. As the wheelset is shifted laterally from its centered position between the rails, rolling radius difference will vary with respect to the lateral location of the point of rolling contact on each wheel.

3.1.7 stability taper: A single weighted value based on a linear representation of the wheel tread taper across the running surface of the wheelset as the wheelset is shifted laterally in both directions from its centered position between the rails.

3.1.8 truck hunting: Unstable dynamic motion comprising sustained oscillations in yaw and lateral displacement of the truck frame and wheelsets from flange to flange.

3.1.9 wheel profilometer: Any device that accurately measures wheel flange and tread profile contour. Such a measurement device shall measure with sufficient precision to enable computation of effective wheel tread taper.

3.1.10 wheel tread taper/conicity: The slope of the wheel tread or running surface relative to the axis of the wheelset. Wheel tread taper may vary with respect to the lateral location of the point of rolling contact as the wheelset is shifted laterally from its centered position between the rails. Tread taper is generally expressed as a ratio of the unit rise per lateral distance; for example, 1:20 (a rise of 1 in 20). Conicity is typically expressed as a decimal value; for example, 0.05. Both taper and conicity represent the same quantities.

3.2 Abbreviations and Acronyms

AAR - Association of American Railroads

NRCC - National Research Council Canada

NF - Narrow flange (wheel)

WF - Wide flange (wheel)

ΔR - Rolling radius difference

λ_s - Stability taper, a weighted average tread taper determined around the centered position of the wheelset

y - Lateral shift of the wheelset geometric center with respect to the track centerline

R_{right} - Rolling radius of the right wheel (measured as the perpendicular distance from the wheel/axle center of rotation to the point of contact with the right rail)

R_{left} - Rolling radius of the left wheel (measured as the perpendicular distance from the wheel/axle center of rotation to the point of contact with the left rail)

$N(y)$ - Normal distribution function, centered about $\bar{y}$

σ - Standard deviation of the normal distribution

$\bar{y}$ - Mean value of lateral shift about which the normal distribution is centered

y_L - Left flange clearance defined as the lateral distance the wheelset can shift from its centered position between the rails until the left wheel flange contacts the left rail

y_R - Right flange clearance defined as the lateral distance the wheelset can shift from its centered position between the rails until the right wheel flange contacts the right rail

4. Definition and Measurement of Wheel Tread Taper (Conicity)

4.1 Wheel Tread Taper and Vehicle Stability

Wheel tread taper is the slope of the tread or running surface of the wheel relative to the axis of the wheelset, sometimes referred to as conicity. Tread taper is generally expressed as a ratio of the unit rise per lateral distance; for example, 1:20 (a rise of 1 in 20). Conicity is typically expressed as a decimal value, for example 0.05. Both conicity and taper represent the same quantities. For any given wheel profile, the slope of the tread is generally not constant but varies with respect to the lateral shift of the wheelset.

The tread taper associated with the profile of a wheel plays a critical role in the dynamic behavior of a vehicle. Increased taper, while improving steering performance in curves, can often lead to increased dynamic instability and truck hunting. Truck hunting causes acoustical noise, poor ride quality, potential fatigue damage to suspension components and to the track, and may also increase the risk of derailment. The truck suspension is designed to resist and damp wheelset dynamics, but there will be a critical speed for the vehicle/truck system above which truck hunting will occur. This critical speed is highly dependent on the effective tread taper for a wheel/rail pair, and should be kept as far above maximum operating speeds as possible.

For a conventional wheelset, the effect of wheel tread taper is directly related to the difference in rolling radius between the left and right wheels of a wheelset at its contact position with the rail. When the wheelset shifts laterally from its centered position between the rails, unless the wheel profiles are cylindrical with zero taper, the rolling radius of the left and right wheels will differ and steering forces will be generated; for each revolution of the wheelset, the wheel having a larger rolling radius will travel a longer distance than the wheel having the smaller rolling radius. The higher the taper, the higher the rolling radius difference becomes as the wheelset shifts laterally. While this is helpful in curves, stability becomes an issue on tangent track. This standard does not apply to trucks or vehicles that are equipped with independently rotating wheels.

4.2 Measurement of Wheel Tread Taper

A measurement of the wheel and rail profiles shall be required to determine the (Y,Z) coordinates of each profile. Rail profiles are used to determine the rolling radius difference for a wheelset when in rolling contact. A sufficient number of points must be measured and in the correct geometric orientation to adequately describe each profile. The rolling radius difference is typically calculated by matching the wheel geometry to the rail geometry in order to determine contact conditions.

Accurate measuring systems with capabilities beyond those of the standard "go/no-go" feeler gauges that are used to monitor flange thickness and fixed tread taper are required to determine stability taper. Contact and non-contact, handheld and in-track measuring systems are currently available.

4.3 Rolling Radius Difference

The rolling radius difference function, ΔR, is defined as the difference in the rolling radius between the left and right wheel of a wheelset as the wheelset is shifted laterally from its centered position between the rails.

$$\Delta R(y) = R_{right}(y) - R_{left}(y) \qquad (1)$$

where:

- ΔR = rolling radius difference function
- R_{right} = rolling radius of the right wheel (measured as the perpendicular distance from the wheel/axle center of rotation to the point of contact with the right rail)
- R_{left} = rolling radius of the left wheel (measured as the perpendicular distance from the wheel/axle center of rotation to the point of contact with the left rail)
- y = lateral shift of the wheelset relative to the track centerline

The rolling radius difference function, ΔR, changes as a function of the lateral displacement, y, of the wheelset, and depends on track gage (e.g. 56” to 57¼”, controlled by 49 CFR §213.323), wheelset back-to-back flange distance (53 3/32” to 53 3/8” for narrow flange wheels; 53” to 53 3/32” for wide flange wheels), and the profile of the left and right rail. The flange clearance when a wheelset is centered between the rails, and the change in the rolling radius of each wheel as a wheelset is shifted laterally from its centered position is illustrated in Figure 1.

As the wheelset shifts laterally between the rails, the points of contact between each wheel tread and rail head change giving rise to a different value of ΔR at each value of lateral shift, y. In general, the relationship between ΔR and lateral shift is not linear, except perhaps for the tread of a new wheel with fixed tread taper. Thus, a graph of rolling radius difference, ΔR, against lateral shift, y, will not be a straight line. The tread taper, which is one-half the slope of this graph at any value of lateral shift, y, will also change accordingly. An illustration of the rolling radius difference function, ΔR, for a symmetric profile pairing is depicted in Figure 2.

In practice, the wheel and rail profiles are defined by co-ordinate pairs or discrete points, and the rolling radius difference function is also defined by discrete points, $(y_i, \Delta R_i)$.

Figure 1 - Rolling Radius Difference Between Each Wheel as Wheelset Shifts Laterally

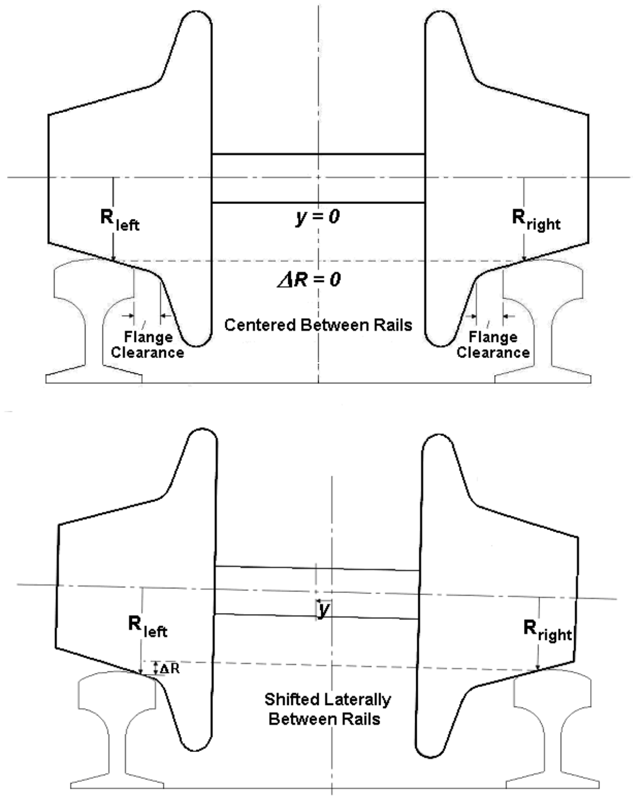
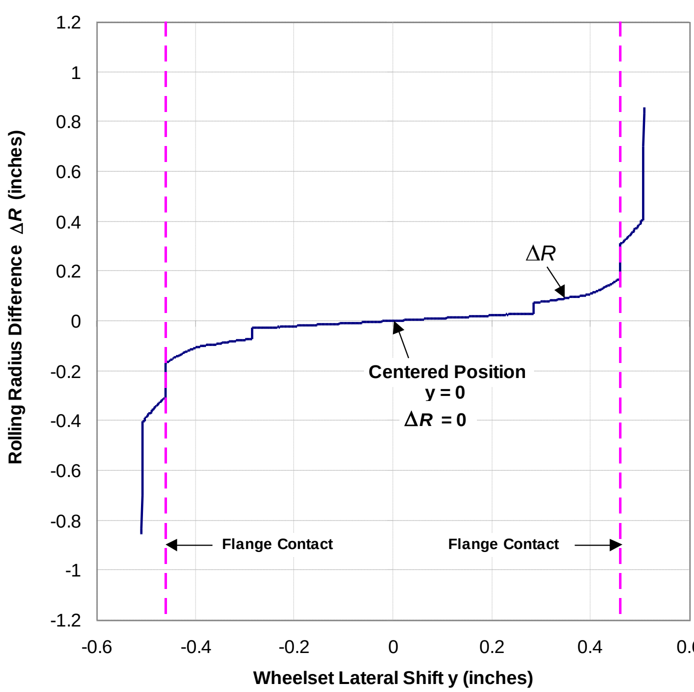


Figure 2 - Example Rolling Radius Difference Curve



4.4 Determination of Stability Taper

Tread taper is generally not a constant value but varies across the running surface of the wheel. A single value of "***stability taper***" is determined to provide a mechanism for obtaining a quantitative assessment of stability performance for a particular set of wheel and rail profiles. The stability taper is derived from the nonlinear rolling radius difference function, ΔR, described in Section 4.2, that requires calculation of the wheel-to-rail contact characteristics for a given combination of wheel and rail profiles with specified wheel flange back-to-back spacing, track gage, and railhead cant angles.

The stability taper, λ_s, is calculated by determining the best representation of the non-linear rolling radius difference function, ΔR, by a straight line weighted to the most-frequented contact area of the tread. Based on testing and vehicle dynamic simulations, a weighting function is defined that uses a normal distribution centered on the most likely or mean value of lateral shift, $\bar{y}$, at which the wheelset operates, and with a standard deviation, σ, defined accordingly. The normal distribution is defined as:

$$N(y) = \frac{e^{-\frac{1}{2}\left(\frac{y-\bar{y}}{\sigma}\right)^2}}{\sqrt{2\pi}\,\sigma} \qquad (2)$$

where: N(y) is the normal distribution function, centered about $\bar{y}$

y is the lateral shift of the wheelset with respect to the track centerline

$\bar{y}$ is the mean value of lateral shift about which the distribution is centered; ($\bar{y} = 0$ on tangent track)

σ is the standard deviation of the distribution

The method of least squared error is applied to determine the best straight-line fit that minimizes the difference between the approximate straight line and the actual non-linear rolling radius difference function at every value of lateral displacement but weighted to the differences at the most-frequented lateral displacements. The stability taper, λ_s, is calculated as one-half the slope of this straight line.

Using the normal distribution as the weighting function centered about the most likely or mean value of lateral shift, $y = \bar{y} = 0$, with a fixed value of σ = 0.1 inches, the stability taper is determined as an integral expression and approximated by a summation of discrete points:

Stability Taper

$$\lambda_s = 50 \int_{-y_L}^{+y_R} N\, \Delta R\, y\, dy \approx 50 \sum_{i=1}^{n-1} N_i y_i \Delta R_i \left(y_{i+1} - y_i \right) \tag{3}$$

This equation is valid only if:
1. The left and right flange clearances, $y_L > 0.3 inches$, $y_R > 0.3 inches$; that is, the flange clearance is greater than 0.3 inches (3σ) whether the wheelset is shifted to the left or to the right from its centered position between the rails.
2. The values of y and ΔR are given in units of inches. If values of y and ΔR are given in units of millimeters, the constant, "50", must be replaced by the constant "0.0775".

where: λ_s is the stability taper

ΔR is the non-linear rolling radius difference function, in units of inches

N is the normal distribution function centered about $\bar{y} = 0$, used as the weighting function

y is the lateral shift of the wheelset with respect to the track centerline, in units of inches

y_L is the left flange clearance defined as the maximum lateral distance the wheelset can shift from its centered position between the rails until the left wheel flange contacts the left rail, in units of inches

y_R is the right flange clearance defined as the maximum lateral distance the wheelset can shift from its centered position between the rails until the right wheel flange contacts the right rail, in units of inches

and, for summation of discrete points:

N_i is a discrete value of the normal distribution function determined at y_i

y_i is a discrete value of wheelset lateral shift with respect to the track centerline, in units of inches

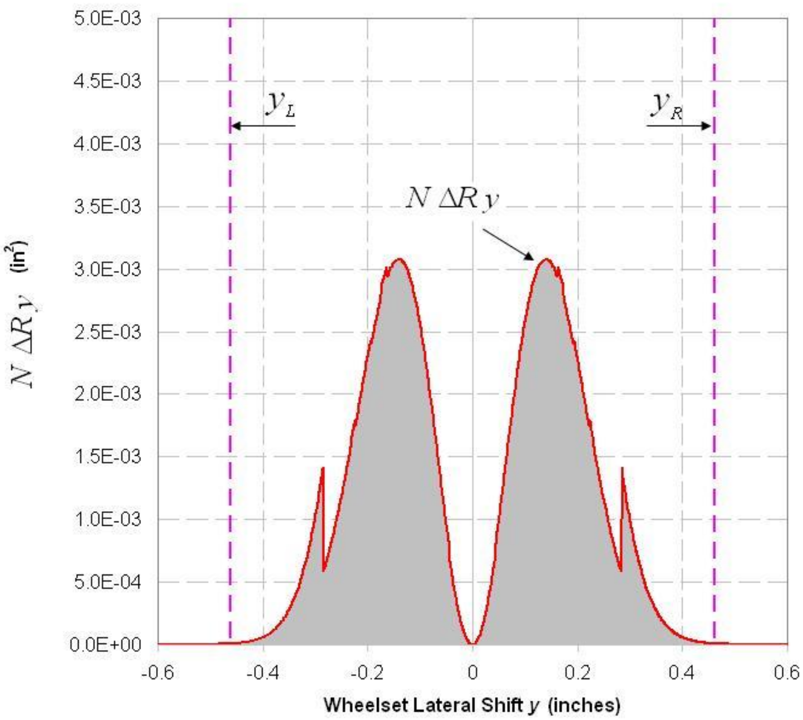
ΔR_i is a discrete value of rolling radius difference at y_i

n is the number of discrete points, $\left(y_i, \Delta R_i \right)$, between $y = -y_L$ and $y = y_R$

For the purposes of this equation, each flange clearance, y_L and y_R, is determined as the maximum lateral distance the wheelset can shift from its centered position between the rails to a point at which the angle of contact between the wheel and rail does not exceed 25 degrees with respect to the wheelset axis.

The integral in Equation 3 represents the area under the curve of the function as shown in Figure 3; that is: $Area_1 = \int_{-y_L}^{y_R} N \, \Delta R \, y \, dy$ and $\lambda_s = 50 \times Area_1$

Figure 3 – Computation of Stability Taper, as 50 Times the Shaded Area



If the left and/or right flange clearances are less than 0.3 inches but are equal and opposite, such that $y_R = y_L$, then the following equation may be used:

Stability Taper (Generalized)

$$\lambda_s = \frac{\int_{-y_L}^{+y_R} N \, \Delta R \, y \, dy}{2\int_{-y_L}^{+y_R} N \, y^2 \, dy} \approx \frac{\left(\sum_{i=1}^{n-1} N_i y_i \Delta R_i \left[y_{i+1} - y_i\right]\right)}{2\left(\sum_{i=1}^{n-1} N_i y_i^2 \left[y_{i+1} - y_i\right]\right)} \tag{4}$$

This equation is valid only if:

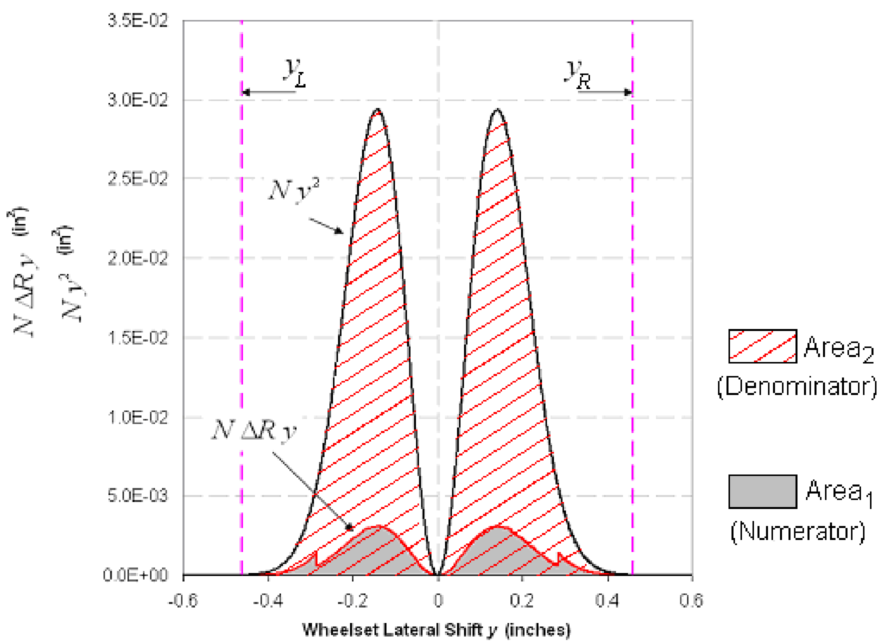
1. $y_R = y_L$, the flange clearance is the same whether the wheelset is shifted to the left or to the right from its centered position between the rails.
2. The values of y and ΔR are given in the same units of distance.

The integrals in the numerator and denominator of Equation 4 represent the area under the curve of the respective functions; that is: $Area_1 = \int_{-y_L}^{+y_R} N \, \Delta R \, y \, dy$ and $Area_2 = \int_{-y_L}^{+y_R} N \, y^2 \, dy$

and Equation 4 represents one-half the ratio of the two areas that are shown in Figure 4:

$$\lambda_s = \frac{1}{2} \frac{Area_1}{Area_2} \qquad (5)$$

Figure 4 – Computation of Stability Taper, As One-Half The Ratio of Areas



Equations (3) and (4) compute the stability taper, λ_s, as an integration or summation over a range of wheelset lateral displacements, y, corresponding to typical contact of a wheelset on tangent track from flange contact on the left rail to flange contact on the right rail. A low decimal value of stability taper is beneficial for stable operation at high speeds. The determination of stability taper is equivalent to computing the weighted least squares fit straight line to the rolling radius difference function over the range of lateral displacements from flange contact to flange contact, and taking one-half the slope of this line.

Informative Annex A gives examples of applying this standard to samole new wheel and rail profiles.

Details of the method used to determine the stability taper and the general derivation are given in the informative Annex B. For interpretation purposes, the straight line representation is equivalent to simplifying the geometry between wheel and rail to the case of conical wheels running on knife-edge rails. The stability taper is then the equivalent taper of a single wheel.

Annex A (informative) Example Application of Standard to a Sample of New Wheel and Rail Profiles

A.1 Stability Tapers for Available Profiles

The *stability taper* definition is intended to assess the contact conditions resulting from combinations of both new and service worn wheel and rail profiles. The results are expected to assist in choosing new wheel profiles and establishing maintenance limits to meet stability requirements of a particular operation.

It is important to note that the results for stability taper can be represented as a decimal value, e.g. 0.05, or as a ratio, e.g. 1:20. The decimal value is typically called "conicity" and the ratio expression is called a "taper." They represent the same quantities however. In this standard, the word "taper" is used because the results are expected to be presented in the ratio form 1:(1/c) where c is actually the decimal result of the expressions (B-11) and (B-12).

Table A-1 provides representative values for the stability taper computed for various combinations of commonly used new wheel and rail design profiles and nominal values of wheel back-to-back distance and track gage. Generally, as the stability taper becomes steeper than 1:10 (corresponding to a decimal value or conicity of 0.1), the risk for truck instability (hunting) at lower vehicle speeds becomes greater and more difficult to control in truck suspension design. Stability tapers of 1:10 to 1:100 are favorable to maintaining higher critical speeds for truck stability.

Table A-1 –Stability Taper – New Wheel and Rail, Nominal and Wide Track Gage

	Track Gage =56.5 (in)				Track Gage =57.0 (in)		
Wheel Profile*	**Rail Profile**	**Flange Clearance (in)**	**Stability**		**Flange Clearance (in)**	**Stability**	
			Conicity	**Taper**		**Conicity**	**Taper**
APTA 140T (Amtrak 1:40 NF) [Tape Line at 2.844 (in)]	140 lb	0.358	0.0256	1 : **39.1**	0.604	0.0258	1:**38.8**
	136 lb	0.398	0.0256	1 : **39.0**	0.644	0.0258	1:**38.7**
	132 lb	0.358	0.0256	1 : **39.1**	0.604	0.0255	1:**39.3**
	119 lb	0.361	0.0256	1 : **39.1**	0.607	0.0258	1:**38.7**
	115 lb	0.326	0.0253	1 : **39.5**	0.572	0.0254	1:**39.4**
APTA 120T (Amtrak 1:20 NF) [Tape Line at 2.844 (in)]	140 lb	0.356	0.0525	1 : **19.1**	0.598	0.0524	1:**19.1**
	136 lb	0.383	0.0528	1 : **18.9**	0.644	0.0525	1:**19.0**
	132 lb	0.356	0.0524	1 : **19.1**	0.598	0.0523	1:**19.1**
	119 lb	0.354	0.0528	1 : **18.9**	0.596	0.0527	1:**19.0**
	115 lb	0.321	0.0525	1 : **19.0**	0.563	0.0524	1:**19.1**
AAR-1B 1:40 WF [Tape Line at 3.063 (in)]	140 lb	0.226	0.0259	1 : **38.6**	0.472	0.0255	1:**39.2**
	136 lb	0.299	0.0273	1 : **36.6**	0.545	0.0255	1:**39.2**
	132 lb	0.236	0.0260	1 : **38.4**	0.482	0.0256	1:**39.0**
	119 lb	0.222	0.2564	1 : **3.9**	0.472	0.0262	1:**38.2**
	115 lb	0.152	0.0268	1 : **37.3**	0.398	0.0254	1:**39.4**
AAR-1B 1:20 WF [Tape Line at 3.063 (in)]	140 lb	0.219	0.0880	1 : **11.4**	0.460	0.0522	1:**19.2**
	136 lb	0.291	0.0614	1 : **16.3**	0.533	0.0522	1:**19.2**
	132 lb	0.222	0.0774	1 : **12.9**	0.464	0.0524	1:**19.1**
	119 lb	0.219	0.3487	1 : **2.9**	0.459	0.0549	1:**18.2**
	115 lb	0.129	0.0630	1 : **15.9**	0.369	0.0527	1:**19.0**
APTA 240 (AAR-1B 1:40 NF) [Tape Line at 3.063 (in)]	140 lb	0.358	0.0259	1 : **38.6**	0.604	0.0258	1:**38.7**
	136 lb	0.401	0.0259	1 : **38.6**	0.647	0.0259	1:**38.6**
	132 lb	0.358	0.0253	1 : **39.6**	0.604	0.0253	1:**39.5**
	119 lb	0.361	0.0258	1 : **38.8**	0.607	0.0258	1:**38.8**
	115 lb	0.326	0.0253	1 : **39.6**	0.572	0.0252	1:**39.7**
APTA 220 (AAR-1B 1:20 NF) [Tape Line at 3.063 (in)]	140 lb	0.341	0.0526	1 : **19.0**	0.583	0.0525	1:**19.1**
	136 lb	0.388	0.0526	1 : **19.0**	0.630	0.0524	1:**19.1**
	132 lb	0.341	0.0525	1 : **19.0**	0.583	0.0524	1:**19.1**
	119 lb	0.339	0.0529	1 : **18.9**	0.581	0.0526	1:**19.0**
	115 lb	0.306	0.0526	1 : **19.0**	0.548	0.0523	1:**19.1**
VIL-15† (NF) [Tape Line at 2.835 (in)]	140 lb	0.379	0.1657	1 : **6.0**	0.627	0.2135	1:**4.7**
	136 lb	0.426	0.1528	1 : **6.5**	0.674	0.1810	1:**5.5**
	132 lb	0.39	0.1964	1 : **5.1**	0.638	0.1882	1:**5.3**
	119 lb	0.403	0.1719	1 : **5.8**	0.650	0.1336	1:**7.5**
	115 lb	0.329	0.1548	1 : **6.5**	0.575	0.2025	1:**4.9**
APTA 320 (NRCC-COM20 NF) [Tape Line at 2.756 (in)]	140 lb	0.340	0.1064	1 : **9.4**	0.582	0.0530	1:**18.9**
	136 lb	0.413	0.0807	1 : **12.4**	0.655	0.0530	1:**18.9**
	132 lb	0.346	0.0961	1 : **10.4**	0.587	0.0532	1:**18.8**
	119 lb	0.351	0.2596	1 : **3.9**	0.594	0.0671	1:**14.9**
	115 lb	0.266	0.2007	1 : **5.0**	0.508	0.0543	1:**18.4**

* Wheel Flange Back-to Back Distance $53\frac{5}{16}$ (in) for NF, $53\frac{1}{16}$ (in) for WF

† Profile used in Europe

Annex B (informative) General Derivation of Stability Taper

B.1 Purpose

This Annex provides background information on the derivation of expressions for stability taper used for assessing wheel/rail contact geometry.

B.2 Derivation of Stability Taper

Wheel/rail interaction is a critical aspect of vehicle performance. An important factor is the geometric compatibility of wheel and rail. The use of tapered wheel profiles allows wheelsets to steer through curves, but also creates a coupling between lateral and yaw motions. This coupling leads to instability (hunting) at high speed. To qualitatively assess the effect of a given wheel/rail pairing, a value for stability taper is derived.

The ability of a wheelset to steer and to remain stable is determined by the rolling radius difference function. This is the difference of the right and left wheel rolling radii as a function of the wheel set lateral position. That is:

$$\Delta R(y) = R_{right}(y) - R_{left}(y) \tag{B-1}$$

The notation, (y), is intended to indicate that all three quantities, rolling radius difference ΔR, right wheel rolling radius, R_{right}, and left wheel rolling radius, R_{left}, are functions of the wheel set lateral displacement, y.

The rolling radius difference typically increases with increasing wheel set lateral displacement. The greater the rolling radius difference, the greater the tendency of the wheel set to steer and return towards a centered position. During curving this is an advantage. The sharper the curve, the greater the difference in distance traveled by the wheel on the outside rail versus the wheel on the inside rail. Increasing the maximum rolling radius difference before flange contact increases the range of curvatures for which pure rolling can be sustained.

Stability on tangent track creates a conflicting requirement. The tendency of a wheelset to steer when displaced laterally results in a coupled lateral and yaw oscillation. Moving towards one rail causes the wheel set to steer towards the other. This is repeated at the opposite rail and a sustained oscillation is established. The wheel set oscillation eventually involves the full vehicle and results in hunting or lateral instability. The greater the rate of change of the rolling radius difference as a function of wheelset lateral position, the lower the speed at which this oscillation reaches a critical amplitude.

Wheel/rail performance objectives for curving and hunting are therefore contradictory and thus difficult to meet using the same profile. The issue is further complicated by the effect of profile changes as both wheels and rails wear. The proposed measure is to quantitatively assess the merits of a specific profile pairing in meeting stability requirements.

The rolling radius difference function is typically calculated using specialized software to determine contact geometry between a wheel set and track element. Critical parameters are the left and right wheel and rail contours, as well as their geometric orientation. For the wheelset, the geometry parameters are the flange back-to-back distance and, in the event of a diameter mismatch, the nominal left and right wheel diameters. For the track element, the geometry parameters are the track gauge and the left and right rail cants. Ideally the calculation should include dynamic worst-case variations in these values.

A sample rolling radius difference function for a symmetric profile pairing is shown in **Figure B1**.

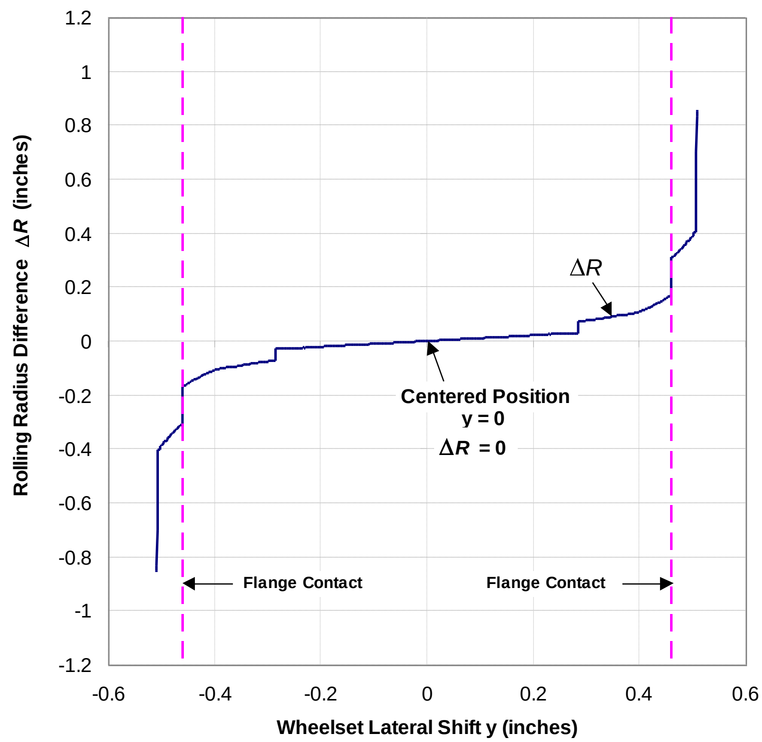


Figure B1 – Example Rolling Radius Difference Data

In the case of an asymmetric pairing, the curve may not pass through the origin. However, this derivation assumes wheelset lateral displacement values have been adjusted such that this is so. To determine a value assessing the stability performance of a profile, the non-linear rolling radius difference function is represented as a linear function of the wheelset lateral displacement. This linear rolling radius difference function, ΔR^{linear}, takes the form of a straight line:

$$\Delta R^{linear} = 2\lambda_s y + \Delta R^{zero} \qquad \text{(B-2)}$$

where λ_s is a parameter called the equivalent conicity or stability taper determined from the slope of the linearized rolling radius difference function (a single weighted average value) and ΔR^{zero} a constant offset equal to the linearized rolling radius difference when $y = 0$. For a centered wheel set and a symmetric profile pairing (i.e., when left and right wheel and rail profiles are identical), $\Delta R^{zero} = 0$. The offset will generally be non-zero for an asymmetric profile pairing, as well as for a wheel set on a constant radius curve.

A linear rolling radius difference function is equivalent to simplifying the geometry between wheel and rail to the case of conical wheels running on knife edge rails. The stability taper or equivalent conicity is then the taper of a single wheel (expressed as a decimal fraction). Equation B-2, or some form thereof, is typically used in assembling linear equations of motion for a wheelset. The stability taper is a measure of the coupling between wheelset lateral and yaw motions. For a given lateral shift, a rolling radius difference is created. The larger this difference (meaning the equivalent conicity is large), the greater the tendency of the wheel set to steer and thus begin to yaw. This single parameter is therefore a useful measure of the influence of the wheel/rail pairing on both vehicle curving and stability.

To linearize the rolling radius difference function to obtain the stability taper, the method of least squared error is used with weighted lateral displacements. The error between the non-linear function (A-1) and the linear approximation (B-2) is as follows

$$\xi = \Delta R - 2\,\lambda_s\, y - \Delta R^{zero} \qquad \text{(B-3)}$$

The squared error is then

$$\xi^2 = \Delta R^2 - 4\,\lambda_s \Delta R\, y - 2\,\Delta R\,\Delta R^{zero} + 4\,\lambda_s^{\;2}\, y^2 + 4\,\lambda_s\, y\,\Delta R^{zero} + (\Delta R^{zero})^2 \qquad \text{(B-4)}$$

An "average squared error" is obtained by applying the above equation for a range of weighted lateral displacements. That is

$$\text{Average Squared Error} = \int_{-y_L}^{+y_R} W(y)\,\xi^2\, dy \quad \text{(B-5)}$$

This expression uses integration to "average" the squared error for lateral displacements ranging from a lower bound $-y_L$ to an upper bound $+y_R$. Values are weighted by the weighting function, W. Equations B-4 and B-5 are used to determine the best fit or least squared error given by the constants, λ_s and ΔR^{zero} that minimize the "average squared error." Two independent equations result leading to the following general expression for the stability taper or equivalent conicity:

$$\lambda_s = \frac{\int_{-y_L}^{+y_R} W\,\Delta R\, y\, dy \int_{-y_L}^{+y_R} W\, dy - \int_{-y_L}^{+y_R} W\,\Delta R\, dy \int_{-y_L}^{+y_R} W\, y\, dy}{2\left(\int_{-y_L}^{+y_R} W\, y^2\, dy \int_{-y_L}^{+y_R} W\, dy - \int_{-y_L}^{+y_R} W\, y\, dy \int_{-y_L}^{+y_R} W\, y\, dy\right)} \quad \text{(B-6)}$$

In practice, the wheel and rail profiles are defined by co-ordinate pairs or discrete points, and the rolling radius difference function is also defined by discrete points, $(y_i, \Delta R_i)$. For uniformly spaced discrete points between $-y_L$ and $+y_R$, Equation A-6 can be rewritten in terms of summations of "n" discrete points as:

$$\lambda_s \approx \frac{\left(\sum_{i=1}^{n} W_i y_i \Delta R_i\right)\left(\sum_{i=1}^{n} W_i\right) - \left(\sum_{i=1}^{n} W_i \Delta R_i\right)\left(\sum_{i=1}^{n} W_i y_i\right)}{2\left[\left(\sum_{i=1}^{n} W_i y_i^2\right)\left(\sum_{i=1}^{n} W_i\right) - \left(\sum_{i=1}^{n} W_i y_i\right)^2\right]} \qquad \text{(B-7)}$$

Equation B-7 may be recognized as one-half the slope of a weighted least squares fit straight line to the discrete points, $(y_i, \Delta R_i)$.

The rate of change of the rolling radius with respect to lateral wheelset position is the critical parameter controlling wheelset and vehicle stability. Theoretical studies indicate that wheel set lateral displacements over measured track geometry are typically normally distributed until instability and severe flange-to-flange contact occur. The appropriate weighting function is thus a normal distribution centered about a mean value of zero. That is:

$$W(y) = N(y) = \frac{e^{-\frac{1}{2}\left(\frac{y}{\sigma}\right)^2}}{\sqrt{2\pi}\,\sigma} \qquad \text{(B-8)}$$

The lower and upper integration bounds ($-y_L$ and $+y_R$) used in Equation B-6 are chosen to represent lateral displacement limits for left and right wheel tread contact. It is proposed that contact angles between the wheel and rail below 25 degrees represent tread contact. At greater angles, it is assumed contact has moved to the flange root or the flange.

In terms of discrete points, $(y_i, \Delta R_i)$, used in Equation B-7, discrete weights are determined as:

$$W_i = N_i = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{y_i}{\sigma}\right)^2} \qquad \text{(B-9)}$$

The assumed distribution of lateral displacements is shown in **Figure B2**. Rolling radius difference values and an example linear fit appropriate for stability are also shown.

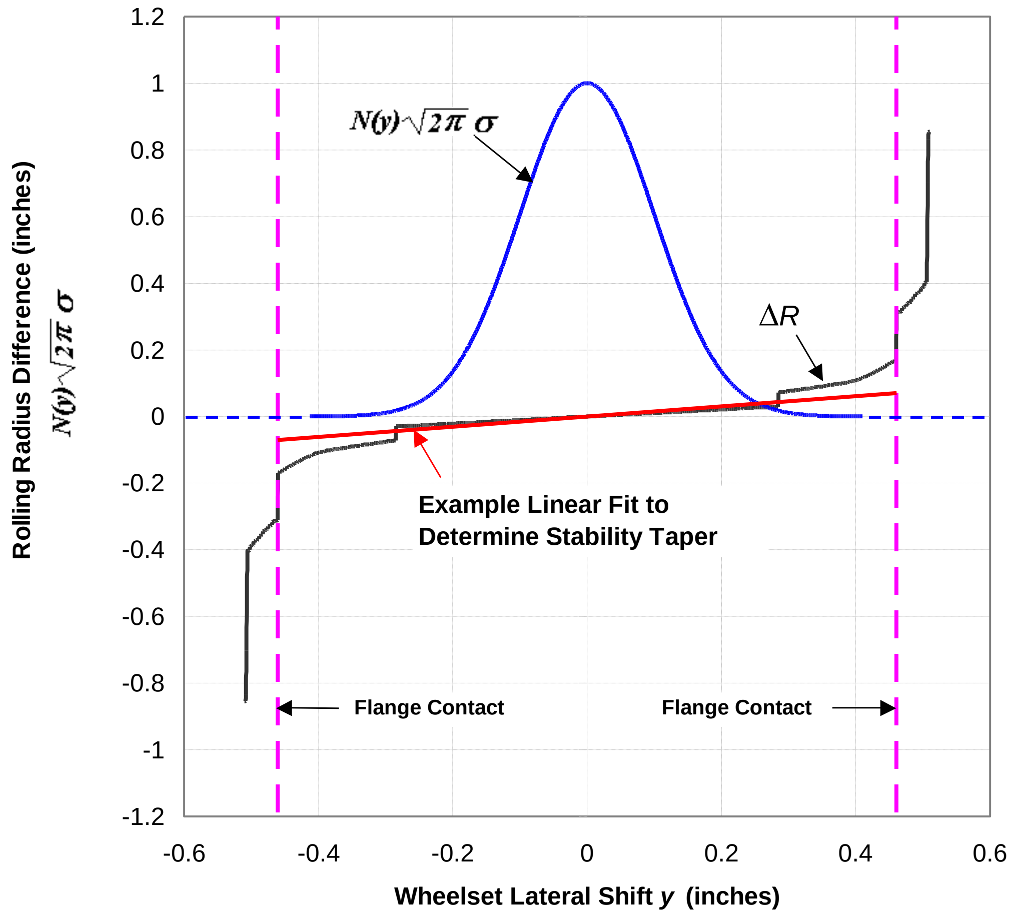


Figure B2. Example Linear Fit for Stability Assessment

Using a normal distribution as the weighting function, Equation B-6 can be simplified if the integration bounds are large relative to the standard deviation, σ. That is:

$$y_R > 3\sigma \text{ and } y_L > 3\sigma \qquad \text{(B-10)}$$

This condition is generally true in the absence of narrow track gage (and thus a small flange clearance) reducing the integration bounds. Then, $\int_{-y_L}^{+y_R} N\; y\, dy \approx 0$.

Equation B-6 can be further simplified by defining a fixed standard deviation. Setting σ as 0.1 inches, then:

Stability Taper

$$\lambda_s = 50\int_{-y_L}^{+y_R} N\ \Delta R\ y\ dy \ \approx\ 50\sum_{i=1}^{n-1} N_i y_i \Delta R_i \left(y_{i+1} - y_i\right) \qquad \text{(B-11)}$$

This equation is valid only if:

1. The integration bounds, $y_R > 0.3 inches$, $y_L > 0.3 inches$; that is, the flange clearance is greater than 0.3 inches (3σ) whether the wheelset is shifted to the left or to the right from its centered position between the rails.
2. The values of y and ΔR are given in units of inches. If values of y and ΔR are given in units of millimeters, the constant, "50", must be replaced by the constant "0.0775".

If the left and/or right flange clearances are less than 0.3 inches but are equal and opposite, such that:

$$y_R = y_L \qquad \text{(B-12)}$$

(this condition is always true for a symmetric profile pairing), then:

Stability Taper

$$\lambda_s = \frac{\int_{-y_L}^{+y_R} N\ \Delta R\ y\ dy}{2\int_{-y_L}^{+y_R} N\ y^2\ dy} \approx \frac{\left(\sum_{i=1}^{n-1} N_i y_i \Delta R_i \left[y_{i+1} - y_i\right]\right)}{2\left(\sum_{i=1}^{n-1} N_i y_i^2 \left[y_{i+1} - y_i\right]\right)} \qquad \text{(B-13)}$$

This equation is valid only if:

1. $y_R = y_L$, the flange clearance is the same whether the wheelset is shifted to the left or to the right from its centered position between the rails.
2. The values of y and ΔR are given in the same units of distance

Note that Equations B-11 and B-13 are based on the normal distribution as defined in Equation B-8. Furthermore, if the conditions defined by B-10 are not met, then the more complex expressions given in Equations B-6 and B-7 must be used.

Annex C Bibliography

[C1] UIC Leaflet 518, "Testing and approval of railway vehicles from the point of view of their dynamic behavior – Safety – Track fatigue – Ride quality", 3rd edition, October 2005.

[C2] UIC Leaflet 519, "Method for determining the equivalent conicity", 1st edition, December 2004.

[C3] Cooperrider, N.K., Law, E.H., Hull, R., Kadala, P.S., Tuten, J.M., "Analytical and Experimental Determination of Nonlinear Wheel/Rail Geometric Constraints", U.S. DOT Report No. FRA-OR&D 76/244, December 1975.

[C4] Magel, Eric, "Development of a Generic Wheel Profile and Matching Rail Profiles for Commuter Systems", CSTT Report #CSTT-HVC-LR-222, June 2005.